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Artificial Intelligence–Assisted Stability-Aware Optimal Control of BWR, HWR, and PWR Nuclear Reactors

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Abstract:

The optimization of a nuclear reactor's performance and stability are two important factors in its optimal control. Stability analysis of a nonlinear system with a high probability of oscillations and bifurcations poses a challenge for the explicit computation of eigenvalues at each time step. Such computations are computationally expensive and nonsmooth near bifurcation points. In this work, a novel optimal control scheme using a neural network is proposed for boiling water reactors (BWR), heavy water reactors (HWR), and pressurized water reactors (PWR). Feedforward neural networks are used to approximate the maximum real eigenvalue of the system's Jacobian as a smooth function of the system's states and bifurcation parameters. For the PWR and HWR systems, a smooth approximation of the hyperbolic tangent function, x/(1+|x|), is used on the network outputs to ensure smoothness in the objective function and constraints for IPOPT optimization. For the BWR systems, the network outputs are used without any further processing because solver stability is not an issue. The approach combines surrogate models within a dynamic optimization problem using Pyomo software, with a soft penalty function and a hard constraint function to prevent unstable solutions. The results show that a positive sign of the penalty parameters is effective for BWR and HWR systems, while a negative sign is effective for PWR systems. The proposed approach is computationally efficient and can be generalized to incorporate stability into optimal control for various nuclear reactor systems.

Key words: bifurcation; optimal control; nuclear reactor; neural network; artificial intelligence

Introduction:

For the safe and efficient operation of nuclear reactors, it is necessary to consider not only performance objectives but also stability. The dynamics of modern reactors, whether boiling water reactors (BWRs), heavy water reactors (HWRs), or pressurized water reactors (PWRs), are nonlinear. These nonlinear dynamics are characterized by various behaviors, including oscillations, multiple steady states, and instability. In particular, under certain conditions, reactors can experience Hopf bifurcations characterized by a change in the sign of the eigenvalue of the system Jacobian located in the positive real axis. Such reactors can experience oscillations that can impact safety and efficiency. Avoiding such critical conditions while maximizing efficiency is a major challenge in the design of nuclear reactors.

The traditional methods for solving stability-constrained optimization problems involve the explicit calculation of eigenvalues at each time step, which can be computationally expensive and even impractical in some cases. Moreover, due to nonsmoothness in the state-space representation of the maximum eigenvalue, symbolic differentiation methods for gradient-based solvers become impractical. To overcome these difficulties, data-driven methods have recently been proposed that provide an alternative solution based on smooth, differentiable representations.

In the present work, feedforward neural networks were used to approximate the maximum real eigenvalue of the reactor system Jacobian as a smooth function of the system state and the bifurcation parameters. A smooth approximation of the hyperbolic tangent function was applied to the neural network outputs when needed.

This was necessary because the neural network outputs were to be used as inputs to the IPOPT solver, and it was important that the first and second derivatives of the function be well-behaved. However, in the case of PWR systems, the neural network outputs were used directly without approximation, since the stability of the IPOPT solver was not a concern.

The proposed framework will incorporate the aforementioned neural network models of stability into the Pyomo-based formulation of the dynamic optimization problem. The soft penalty method and the hard constraint method will be employed to address the bifurcation avoidance. The soft penalty method will be based on incorporating a differentiable function into the objective function. On the other hand, the hard constraint method will be based on imposing strict limits on the eigenvalues. By applying the aforementioned techniques to the BWR, HWR, and PWR models, the flexibility of the proposed method will be demonstrated. The proposed method will be an efficient tool for optimal reactor control. This work will be an important contribution to the development of an efficient and generalizable neural network-based framework for the optimal control of complex nuclear systems.

Literature Review:

Boiling Water Reactors (BWR)

A great deal of literature has been published on nonlinear dynamics and oscillatory instabilities in BWRs, especially regarding Hopf bifurcations and self-sustained power oscillations. March-Leuba et al. (1986) analyzed the nonlinear dynamics and stability of BWRs, focusing on the significance of feedback mechanisms. Munoz-Cobo et al. (1991) extended this work by applying Hopf bifurcation theory and variational methods to investigate the appearance of limit cycles in BWR systems. Wang et al. (1992) analyzed the mathematical structure of nonlinear BWR models, while Tsuji et al. (1993) carried out stability analyses using bifurcation theory. Farawila (1998) analyzed the oscillatory behavior of BWRs by applying modal neutron kinetics.

Subsequent studies focused on the coupled neutronic and thermal-hydraulic effects that cause density-wave oscillations and nonlinear instabilities. In particular, the effects of void distribution parameters and axial power distributions on BWR bifurcation characteristics were studied by Van Bragt et al. (2000). Bifurcation analysis for density wave oscillations using a drift flux model was conducted by Dokhane et al. (2002), and Zhou et al. (2002) used bifurcation analysis for the study of nuclear-coupled BWR dynamics. A nuclear-coupled thermal-hydraulic nonlinear stability analysis using a reduced-order model was proposed by Dokhane et al. (2003), and Zhou et al. (2004) studied the nonlinear dynamics of a reduced-order natural circulation BWR system. Zhou et al. (2005) studied in-phase and out-of-phase oscillations and the effects of azimuthal asymmetry.

Later works continued to investigate nonlinear oscillations and Hopf bifurcation phenomena in BWR systems. Farawila et al. (2006) investigated nonlinear oscillations and limit cycle behavior in BWRs. Rizwan-Uddin et al. (2006) showed that turning points exist in BWR systems and that sub- and supercritical bifurcations occur in simplified BWR systems. Durga Prasad et al. (2008) investigated nonlinear dynamics in natural circulation BWRs. Recent works in BWR stability analysis include reduced-order stability models presented by Lange et al. (2011) and modeling assumptions for BWR stability domains by Bindra et al. (2014). Pandey and Singh (2016)[performed a bifurcation analysis of boiling water reactor on large domain of parametric space

Heavy Water Nuclear Reactors (HWR)

Oscillatory instabilities and bifurcations have also been investigated in advanced heavy-water reactor systems. Nayak et al. (1998) carried out a linear analysis of thermal hydraulic instabilities occurring in HWR systems. Nayak et al. (2001) have also carried out an investigation of thermal hydraulic instabilities occurring in natural circulation heavy water-moderated boiling light water-cooled reactors with the influence of delayed neutrons. Garg (2005) conducted a parametric study of thermal-hydraulic instabilities in AHWR systems.

Subsequent research has also shown nonlinear behavior and the occurrence of Hopf bifurcation in such systems. Kovelamudi et al. (2007) investigated Hopf bifurcation and limit-cycle oscillations in HWR systems. Wahi et al. (2011) conducted a nonlinear stability analysis and a parametric study of HWR systems. Pandey et al. (2015) conducted an extensive study of bifurcations in a simplified HWR system. In subsequent research, the study has been extended to large parametric domains by the same authors.

Pressurized Water Reactors (PWR) and General Reactor Dynamics

Several studies have also been conducted on nonlinear dynamics and oscillations in various models of pressurized water reactors (PWRs) and nuclear reactor kinetics. Merk et al. (2005) derived various time-scale expansions for neutron kinetics, applicable to point kinetics models in many reactor systems, such as PWRs. Sinha et al. (2006) presented an innovative nuclear reactor model fueled with thorium, while Gabor et al. (2009) developed models of nuclear reactor dynamics that include temperature feedback and xenon poisoning effects. Nahla (2009) derived an analytical solution concerning point reactor kinetics equations with delayed neutron effects and adiabatic feedback. Pirayesh et al. (2016) studied local bifurcations in nuclear reactor dynamics using Sotomayor's theorem, which provides additional theoretical results on nuclear reactor dynamics.

Research Gap and Motivation for the Present Work

While substantial progress has been made in understanding nonlinear dynamics, bifurcation phenomena, and oscillatory instabilities in nuclear reactor systems, the majority of the existing literature has focused on identifying the Hopf bifurcation point and analyzing limit cycle dynamics. Such analyses have been useful in understanding the underlying phenomena of the reactor instabilities in BWR, AHWR, and PWR systems. However, relatively few studies have been devoted to the active avoidance or control of Hopf bifurcations using optimal control approaches. For instance, while numerical bifurcation analysis packages such as MATCONT are extremely efficient at identifying the Hopf bifurcation point, they are rarely used in conjunction with optimal control approaches for controlling the dynamics of nuclear reactors.

The present work proposes bifurcation analysis using the MATCONT package and the PYOMO optimal control package. For instance, the Hopf bifurcation point that gives rise to limit cycles will be identified, and the information obtained will be used to formulate an optimal control problem using a penalty function to avoid undesirable limit-cycle dynamics.

Model Equations Boiling Water Reactor (BWR) model (Pandey et al (2016)

The variables involved are (nv(t),cv(t); tv(t); $\rho a(t)$; $\rho t(t)$) and they represent the excess neutron population, excess population of delayed neutron precursors, excess fuel temperature, excess void reactivity feedback, and the derivative of the excess void reactivity feedback with time. All the parameters have been obtained from the nuclear properties and the reactor's geometry. The model equations are

$$\frac{d(nv)}{dt} = 2\frac{\rho a}{\lambda c} + 2d(\frac{tv}{\lambda c}) - \frac{beta}{\lambda c} + \lambda s(cv)$$
$$\frac{d(cv)}{dt} = \frac{beta}{\lambda c}nv - \lambda c(cv)$$
$$\frac{d(tv)}{dt} = a1(nv) - a2(tv)$$
$$\frac{d(\rho a)}{dt} = \rho t$$
$$\frac{d(\rho t)}{dt} = k(tv) - a3(\rho t) - a4(\rho a) \quad (1)$$

The base parameters are a1=25.04; a2=0.23; a3=2.25; a4= 6.82; k=-3.7e-03; d=-2.52e-05; β =0.0056; λc =4.e-05; λs =0.08.

Details can be found in Pandey and Singh (2016).

Heavy Water Nuclear Reactor Model Equations (Wahi et al, (2011)

In this model, xv1, xv2, xv3, and xv4 represent fluctuations around the steady-state operating values of the neutron density, delayed neutron precursor density, average fuel temperature, and the coolant void fraction, respectively, and t represents a non-dimensional time. The parameters af and av are, respectively, proportional to the fuel temperature coefficient of reactivity and the void coefficient of reactivity. The non-dimensional parameters b, ppar, and qpar represent combinations of the parameters (the neutron generation time, delayed neutron fraction, heat capacities, and coolant temperature).

The model equations are

$$\frac{d(xv1)}{dt} = -xv1 + xv2 + af(xv3) + av(xv4) + af(xv1)xv3 + av(xv1)xv4$$
$$\frac{d(xv2)}{dt} = b(xv1 - xv2);$$
$$\frac{d(xv3)}{dt} = ppar(xv1 - xv3)$$
$$\frac{d(xv4)}{dt} = qpar(xv3) \quad (2)$$

ppar is used as the bifurcation and control parameter. The parameter values are af = -1.5,

av = -250, b = 0.0055, and qpar = 0.03. These parameters are scaled and non-dimensional.

Model equations Pressurized Water Nuclear Reactor(Pirayesh et al (2016)

The model equations are

$$\frac{d(pv)}{dt} = \frac{pv}{\lambda c}\left(\alpha f(uv) + \alpha c(wv) + c0 - \beta\right) + \frac{\beta cv}{\lambda c}$$
$$\frac{d(cv)}{dt} = \lambda s(pv - cv)$$
$$\frac{d(uv)}{dt} = a1(pv) - a2(uv - wv + \phi 1)$$
$$\frac{d(wv)}{dt} = a3(uv - wv + \phi 1) - a4(wv + \phi 2) \quad (3)$$

The base parameter values are

a1=95.057; a2=0.251; a3=0.092;a4=2.749; $\phi 1$ 1=395; λc =0.00002; β =0.006019;

λs =0.15; αf =-0.0000324; αc =-0.000213; $\phi 2$ =15; c0=0.004.

pv, cv, uv, and wv represent the relative power of a reactor, the relative concentration of delayed neutron precursors, the fuel temperature, and the coolant temperature. λc is the effective generation time of neutrons, c0 is the reactivity inserted into the reactor core, β is the delayed neutrons fraction, λs is the effective precursor radioactive decay constant, αf is the reactivity coefficient of fuel temperature, αc is the reactivity coefficient of coolant temperature. a1, a2, a3, a4, are the thermo- hydraulics coefficients

Bifurcation analysis and Optimal Control

Bifurcation Analysis

Bifurcation calculations are performed using the MATLAB software MATCONT. Bifurcation analysis explains the main causes for multiple steady-states and limit cycles. Branch points and limit points cause multiple steady-state solutions while limit cycles and oscillatory behavior are causd by Hopf bifurcation points. The MATLAB program that effectively locates limit points, branch points, and Hopf bifurcation points is MATCONT. This program was developed and improved by several researhers (Dhooge Govearts, and Kuznetsov, (2003); Dhooge Govearts, Kuznetsov, Mestrom and Riet, (2004)). This program is very effective in identifying Limit points(LP), branch points(BP), and Hopf bifurcation points(H) for an system of ordinary differential equations

$$\frac{dx}{dt} = f(x,\alpha) \quad (4)$$

$x \in R^n$ where the bifurcation parameter is α . The gradient vesctor is orthogonal to the tangent and hence the tangent plane at any point $w = [w_1, w_2, w_3, w_4, ...w_{n+1}]$ must satisfy

$$Aw = 0 \quad (5)$$

The matrix *A* is defined by

$$A = [\partial f / \partial x \quad | \partial f / \partial \alpha] \quad (6)$$

The sub-matrix $\partial f / \partial x$ is the Jacobian matrix. For both limit and branch points, the Jacobian matrix $J = (\partial f / \partial x)$ must have a determinant of 0.

At a limit point, the *n+1* *th* component of the tangent vector w_{n+1} = 0 . For a branch point, the matrix $B = \begin{bmatrix} A \\ w^T \end{bmatrix}$ must be singular and have a determinant of 0.

At a Hopf bifurcation point,

$$\det(2 f_x(x,\alpha) @ I_n) = 0 \quad (7)$$

@ indicates the bialternate product while I_n is the n-square identity matrix. Hopf bifurcations cause limit cycles and should be eliminated because limit cycles make optimization and control tasks very difficult. More details can be found in Kuznetsov (1998); 2009) and Govaerts [2000] respectively.

Optimal Control

Pyomo.dae (Hart et al, 2017) is used for the Optimal Control calculations. Pyomo.DAE is a powerful extension of the Pyomo optimization modeling framework, which is well-suited for solving dynamic systems of differential and algebraic equations. It is a symbolic environment for solving differential-algebraic equation systems in the context of optimization problems. This is very important in process systems engineering, chemical kinetics, and control systems, where the dynamic response of systems is of prime interest.

At its heart, Pyomo.DAE enables users to define time-varying variables, derivatives, and constraints symbolically, which can be easily integrated into a Pyomo model. Users can easily define continuous sets for time or other continuous variables, which can be used to define their derivatives over those sets. This symbolic approach enables users to easily discretize continuous differential-algebraic equation systems using finite difference, collocation, or orthogonal collocation methods, thereby transforming continuous differential equations into algebraic equations that can be solved with standard solvers. The framework can handle both initial-value problems and dynamic optimization problems. In dynamic optimization, Pyomo.DAE allows the formulation of time-dependent objective functions and constraints, which is particularly useful in optimal control, energy systems, and chemical process scheduling problems.

One of the major advantages of Pyomo.DAE is that it is compatible with the Pyomo ecosystem. This allows users to leverage existing solver interfaces, variable bounds, nonlinear constraints, and objective functions within a combined static and dynamic modeling framework. Furthermore, the symbolic framework makes it easier to perform model verification, automatic differentiation, and sensitivity analysis. Pyomo.DAE provides a flexible, extensible, and open-source environment for modeling, simulation, and optimization of dynamic systems. By integrating symbolic modeling of DAEs with powerful discretization and optimization capabilities, it provides a unique framework for solving complex time-dependent problems. Its tight integration with Pyomo enables the efficient solution of both simple and complex dynamic optimization problems, making it a cornerstone of modern computational modeling of dynamic systems. In Pyomo.DAE , the differential equations are converted to a Nonlinear Program (NLP) using the orthogonal collocation method. The NLP is solved using IPOPT (Wächter & Biegler, 2006)

Formation of stability Dataset from MATCONT results

A stability dataset was developed based on the results from numerical continuation calculations carried out in MATCONT. The stability dataset consists of rows, each representing a continuation point from an equilibrium branch. The columns in each row consist of the state variables, the bifurcation parameter, and a stability measure. The stability measure is a numerical value derived from the Jacobian matrix. The Jacobian matrix is computed numerically at each equilibrium point. The eigenvalues are then computed automatically using MATLAB. The maximum value of the real part of these eigenvalues is then computed as a scalar stability measure.

The stability measure is computed using "eig_real_max = max(real(eigvals));" in MATLAB. The stability measure is a quantitative metric in which negative values indicate locally asymptotically stable equilibria, positive values indicate instability, and a zero crossing indicates a Hopf bifurcation. The stability dataset is then saved as a CSV file. The dataset can then be used in subsequent computational calculations to perform classification or regression to identify stability boundaries or approximate bifurcations.

Neural Network Surrogate for Stability Prediction

Direct embedding of eigenvalue calculations into IPOPT-based optimal control is impractical for several reasons: (i) computing eigenvalues at each time step is computationally expensive, (ii) the mapping from states to the maximum eigenvalue is non-smooth near eigenvalue crossings, and (iii) symbolic differentiation of eigenvalues is challenging.

Prior to neural network training, all input variables were standardized to improve numerical conditioning and training stability. Let x_{raw} denote the vector of state variables and bifurcation parameter obtained from the stability dataset. For each input variable j, the training mean μ_j and training standard deviation σ_j were computed over the training dataset as the arithmetic mean and standard deviation, respectively. The training mean for the input variable j is defined as the arithmetic average over all training samples as $\mu_j = \frac{1}{N}\sum_{k=1}^{N} x_j^{(k)}$ and the training standard deviation for the input variable j is defined as: $\sigma_j = \frac{1}{N}(\sum_{k=1}^{N} x_j^{(k)} - \mu_j)^2$

The standardized inputs were defined as

$$x_j = \frac{x_{\text{raw},j} - \mu_j}{\sigma_j}.$$

This transformation ensures that each input variable has zero mean and unit variance over the training set, thereby improving neural network conditioning and gradient-based optimization performance.

The vectors μ, σ μ and σ computed during training were stored and embedded identically within the Pyomo optimal control formulation to ensure consistency between neural network training and deployment.

To overcome these limitations, a feedforward neural network is trained to approximate the maximum eigenvalue as a smooth function of the system state and bifurcation parameter. A typical architecture employs the hyperbolic tangent (tanh) as a smooth activation function. If the input vector is denoted by x, which are the scaled variables, then the network is defined as:

$$\begin{aligned} z1 &= tanh\left(W1\, x + b1\right) \\ z2 &= tanh\left(W2\, z1 + b2\right) \\ \lambda\_max\_NN &= W3\, z2 + b3 \end{aligned} \quad (8)$$

Because tanh is infinitely differentiable, the network is fully smooth, guaranteeing the availability of first and second derivatives required by IPOPT. Here, W1, W2, and W3 are the weights that scale and combine inputs or hidden-layer features, while b1, b2, and b3 are the biases that allow the neurons to shift their activation independently of the inputs. Without biases, the network output would be constrained to pass through the origin, limiting flexibility. The neural network may use a smooth activation function such as hyperbolic tangent (tanh), or a smooth tanh approximation defined by $\tanh_{approx}(x) = \frac{x}{1+|x|}$ if required for solver compatibility.

The hidden-layer outputs z1, z2, represent nonlinear combinations of the inputs and previous-layer features, respectively. Each element of z_1 is a smoothed combination of the original inputs, while each element of z2 encodes more abstract patterns extracted from z1. The final output λ_max_NN provides a smooth approximation of the maximum real eigenvalue, enabling efficient and differentiable stability evaluation within the optimal control problem. Fig. 1 shows a chart describing the computational strategy.

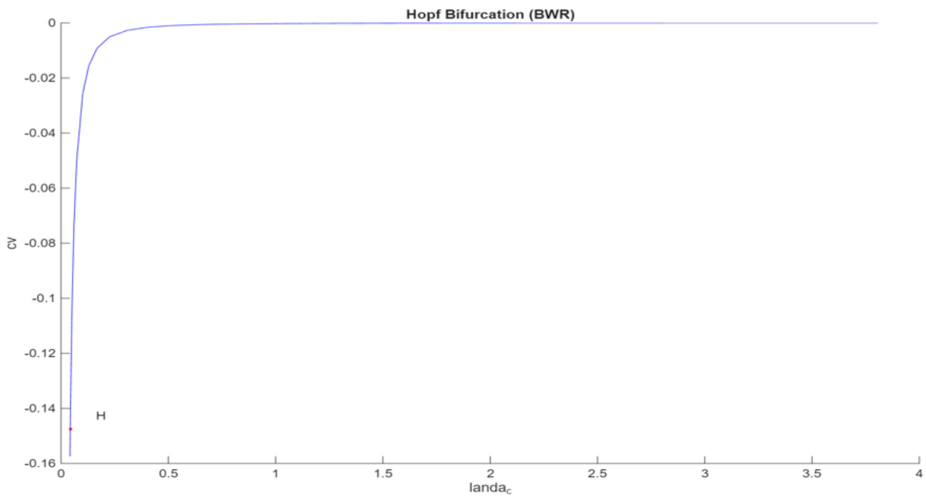

Figure 1: *a Hopf bifurcation for BWR*

The integration into optimal control is done in two ways a) Soft penalty formulation and b) a Hard constraint formulation. formulation For the soft constraint formulation, we use a smooth_max function that converts λ into a smooth, nonnegative penalty that only "activates" when the system is unstable:

$$s_{\max}(\lambda+\varepsilon) = smooth-\max(\lambda+\varepsilon) = \frac{(\lambda+\varepsilon)+\sqrt{(\lambda+\varepsilon)^2+\varepsilon}}{2} \quad (9)$$

λ is the neural network's predicted maximum eigenvalue at the current state and parameter, while ε is a small positive safety margin to ensure differentiability. The soft penalty formulation involves the new objective function, where the original objective function $(\sum P(t))^2$ is modified to $\sum(P(t)-\alpha.s_{\max}(\lambda+\varepsilon))^2$. When no measures are taken, $(\sum P(t))^2$ is optimized and when the soft penalty formulation is integrated, $\sum(P(t)-\alpha.s_{\max}(\lambda+\varepsilon))^2$ is the objective function. In both problems. α controls how aggressively instability is penalized, and ε prevents numerical issues at exactly λ = 0 and slightly shifts the stability boundary.

For the Hard constraint formulation, an additional constraint $\lambda\_max\_NN(t) \leq 0$ is added to the original constraints. This strictly prohibits unstable trajectories.

In the optimal control of nonlinear biochemical processes, the hard Hopf constraint is frequently difficult to apply in the sense that the maximal real eigenvalue of the system Jacobian matrix must stay non-positive at all times. However, the process dynamics and constraints may require the system to exhibit transient behavior in the region of mild instability in order to minimize the objective function. The hard Hopf constraint may prevent the nonlinear solver from converging in such cases because the trajectory may become infeasible. To extend the hard Hopf formulation and alleviate the aforementioned limitations, we propose using a soft Hopf penalty function. The penalty function is directly included in the objective function and is differentiable. The application of the soft Hopf penalty function ensures the generation of a physically realistic and feasible trajectory. The gradients of the objective function are well-behaved for the IPOPT solver, making the proposed approach computationally efficient for the optimal control of nonlinear biochemical processes.

Results and Discussion:

For the BWR problem, when λc is the bifurcation parameter, a Hopf bifurcation point are found at $(nv,\ cv,\ tv,\ \rho a,\ \rho t, \lambda c)$ values of (-0.049444, -0.147982, -5.382997, 0.00292, 0 0.043256) (Fig 1a). The limit cycle created by this Hopf bifurcation is shown in Fig. 1b.

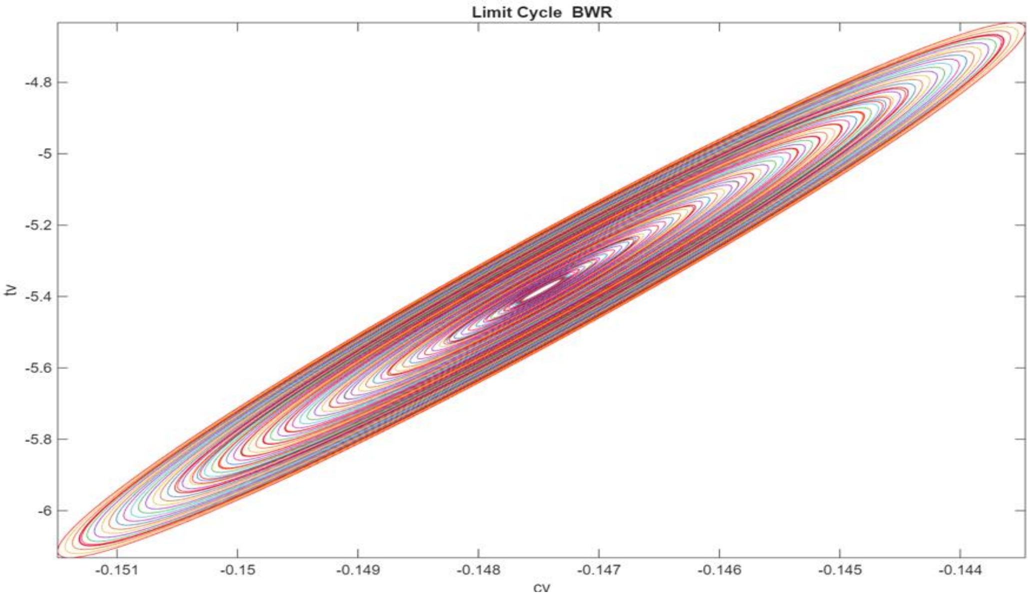

Figure 1b: *Limit Cycle BWR*

For the optimal control, $\sum_{t_{i=0}}^{t_i=t_f}(cv(t_i))$ was minimized. Initially, no measures were taken to avoid the Hopf bifurcations The obtained value of $\sum_{t_{i=0}}^{t_i=t_f}(cv(t_i))$ was -1.149. Subsequently, $\sum_{t_{i=0}}^{t_i=t_f}(cv(t_i))$ was minimized ($(\alpha = 1)$ using the soft Hopf penalty formulation, and the obtained value of $\sum_{t_{i=0}}^{t_i=t_f}(cv(t_i))$ was -2.0324. In this case, a more beneficial outcome resulted from avoiding Hopf bifurcation points. In the BWR problem, the network was used directly without additional smoothing, as solver compatibility was not an issue. Figure 2a -2d shows the optimal control profiles. The control

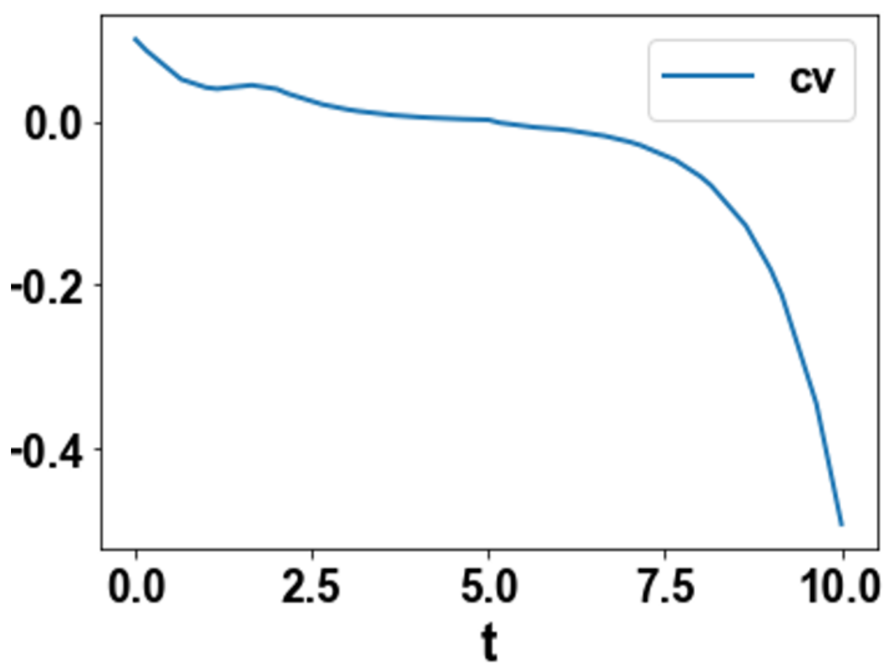

Figure 2a: *optimal control ( )*

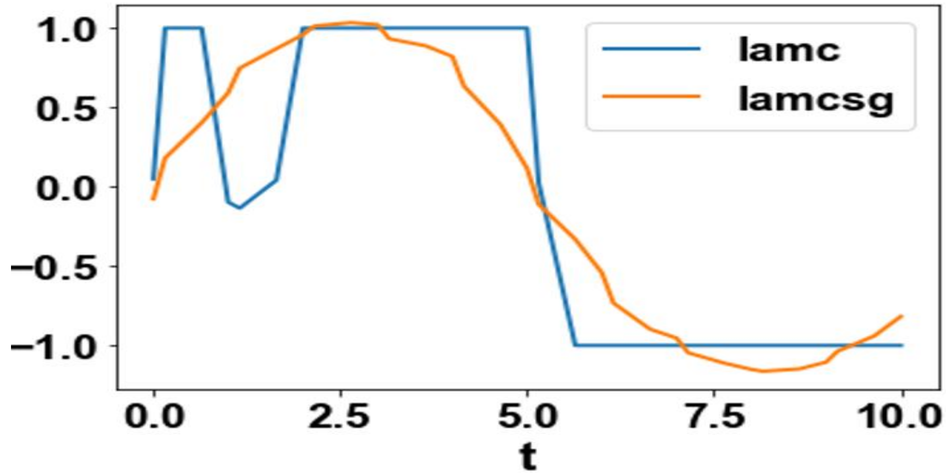

Figure. 2b: *control profiles* $\lambda_C, \lambda_C sg$ *(* $\alpha = 0$ *)*

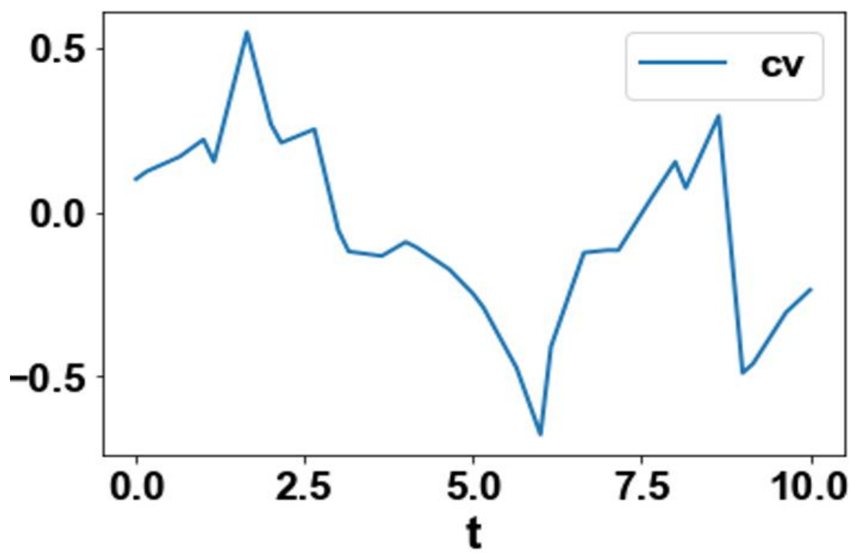

Figure 2c: *optimal control cv profile (* $\alpha = 1$ *)*

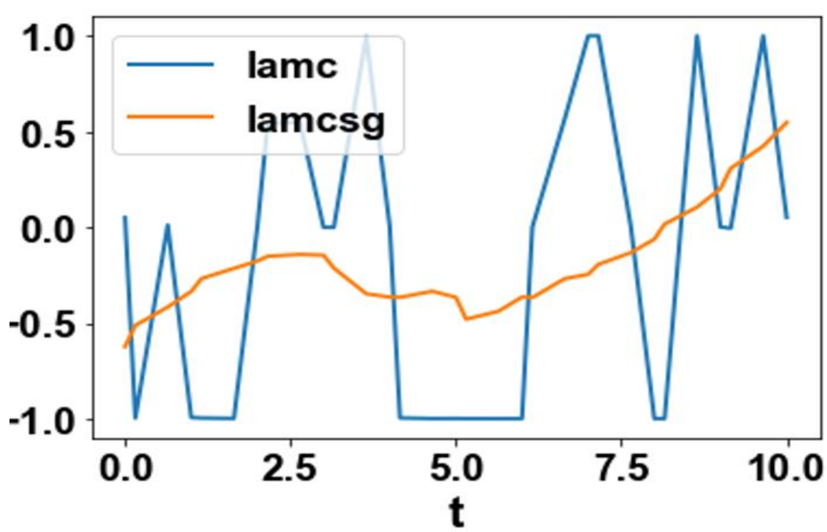

Figure 2d : *optimal control profiles* $\lambda_C, \lambda_C sg$ *(* $\alpha = 1$ *)*

profile exhibits spikes, and this was remedied using the Savitzky-Golay filter to produce the smooth control profile $\lambda_c sg$.

In the Heavy water problem, ppar is the bifurcation parameter, and a Hopf bifurcation point was obtained at $(xv1, xv2, xv3, xv4, ppar)$ values of (0; 0; 0; 0; 2.001113) (Fig. 3a).). The limit cycle created by this Hopf bifurcation is shown in Fig. 3b.

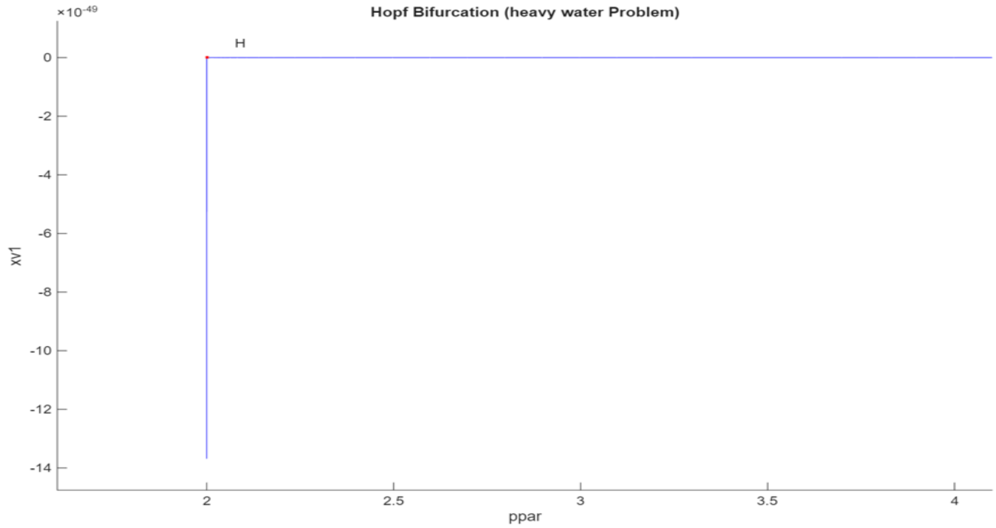

Figure 3a*: Hopf bifurcation for Heavy Water Nuclear Reactor Problem*

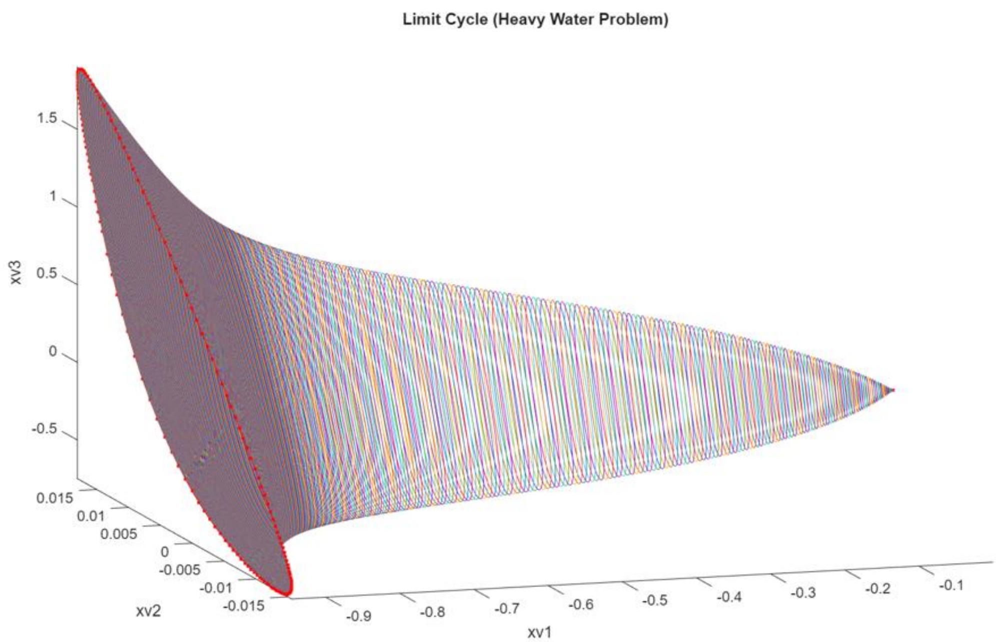

Figure 3b : *Limit Cycle for Heavy Water Nuclear Reactor Problem*

For the optimal control, $\sum_{t_{i=0}}^{t_i=t_f}(xv1(t_i))$ was minimized. Initially, no measures were taken to avoid the Hopf bifurcations. The obtained value of $\sum_{t_{i=0}}^{t_i=t_f}(xv1(t_i))$ was --29.576. Subsequently, $\sum_{t_{i=0}}^{t_i=t_f}(xv1(t_i))$ was minimized $(\alpha = 50)$ using the soft Hopf penalty formulation, and the obtained value of $\sum_{t_{i=0}}^{t_i=t_f}(xv1(t_i))$ was -82.895. In this case, a more beneficial outcome resulted from avoiding Hopf bifurcation points. Figures 4a -4f show the optimal control profiles. In the heavy water nuclear reactor problem, the neural network used a smooth activation a smooth tanh approximation defined by $\tanh_{approx}(x) = \frac{x}{1+|x|}$.

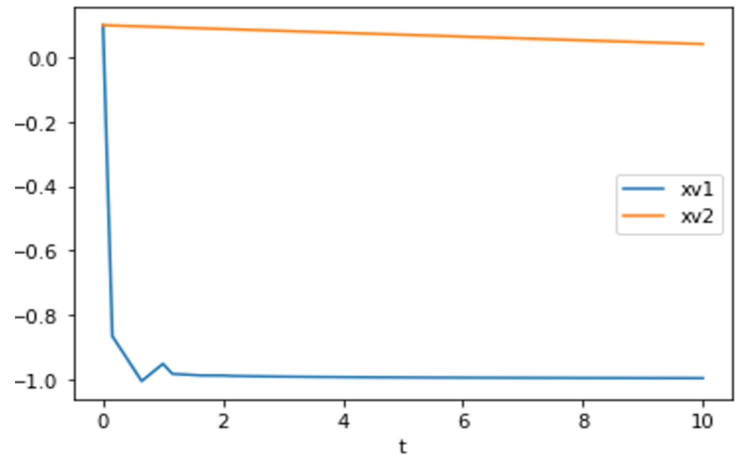

Figure 4a :_optimal control (Heavy Water Nuclear Reactor) xv1 xv2 profiles_

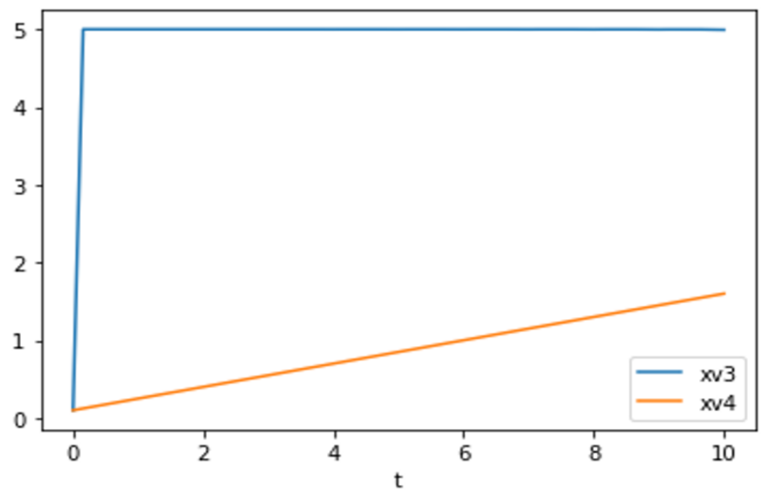

Figure 4b: _optimal control (Heavy Water Nuclear Reactor) xv3 xv4 profiles_ $(\alpha = 0)$

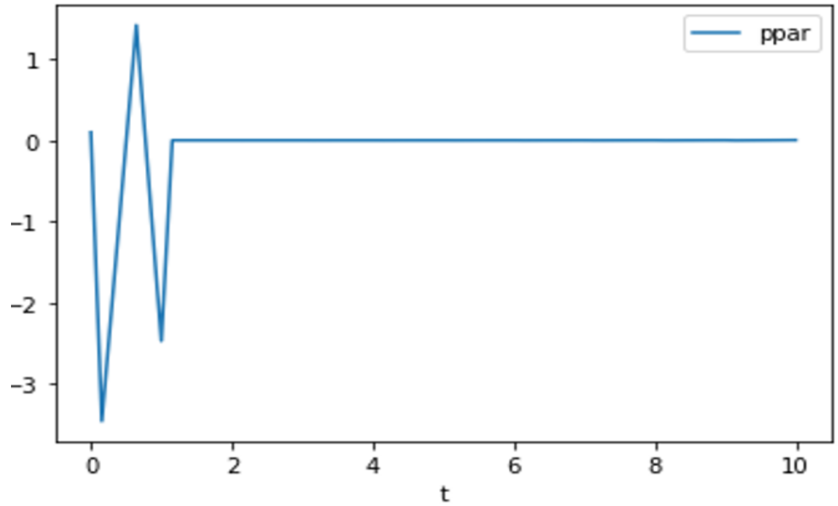

Figure 4c: _optimal control (Heavy Water Nuclear Reactor) ppar profile_ $(\alpha = 0)$

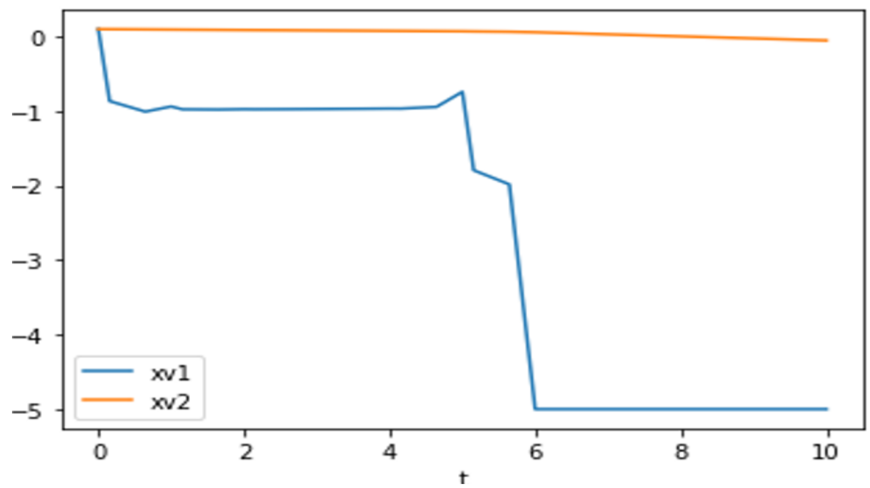

Figure 4d: *optimal control(Heavy Water Nuclear Reactor) xv1 xv2 profiles* $(\alpha = 50)$

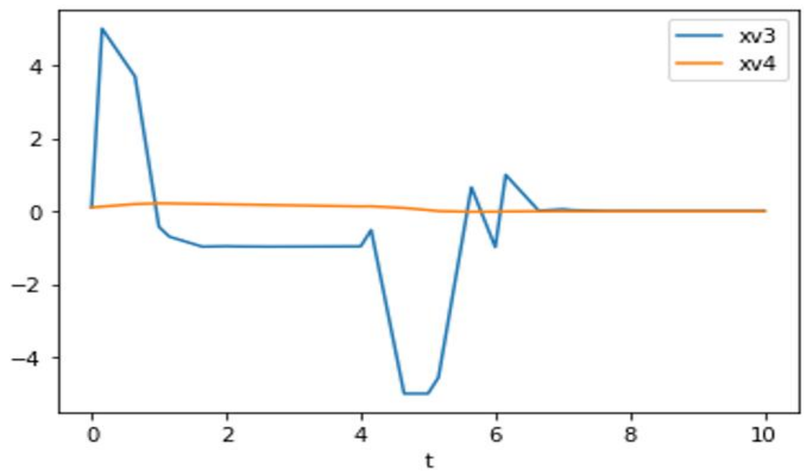

Figure 4e: *optimal control (Heavy Water Nuclear Reactor) xv3 xv4 profiles* $(\alpha = 50)$

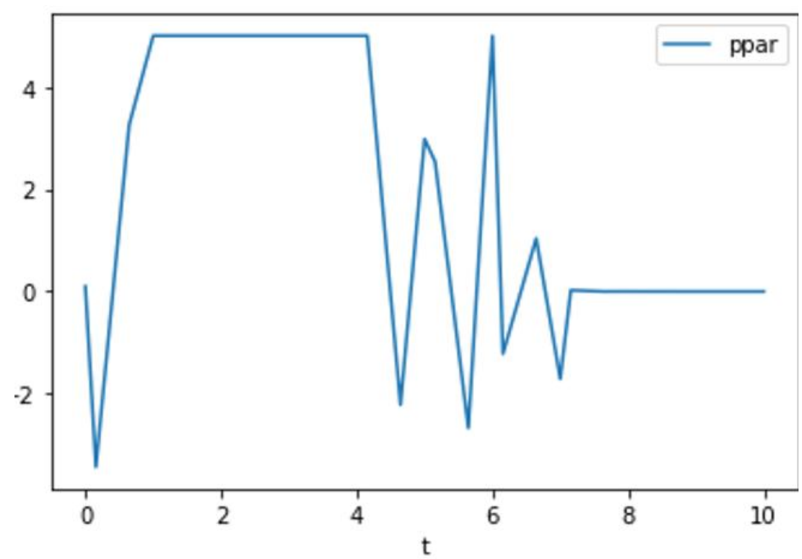

Figure 4f: *optimal control (Heavy Water Nuclear Reactor) ppar profile* $(\alpha = 50)$

In the Pressurized water reactor (PWR) problem, λ_S is the bifurcation parameter, and a Hopf bifurcation point was obtained at $(pv, cv, uv, wv, \lambda_S)$ values of (0; 0; 0; 0; 2.001113) (Fig. 5a).). The limit cycle created by this Hopf bifurcation is shown in Fig. 5b.

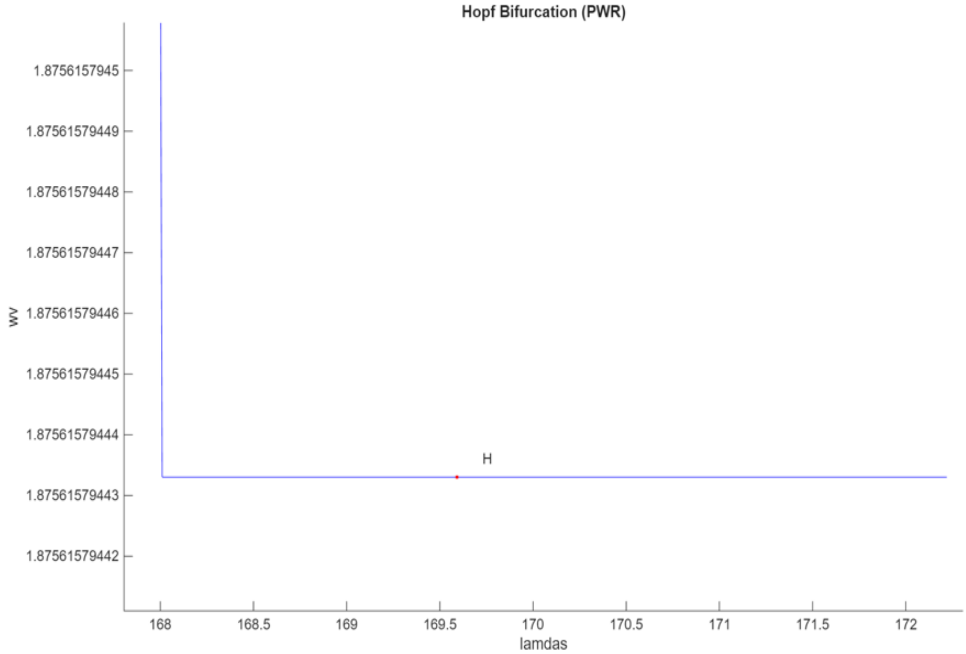

Figure 5a: *(Hopf Bifurcation (pressurized water reactor (PWR)*

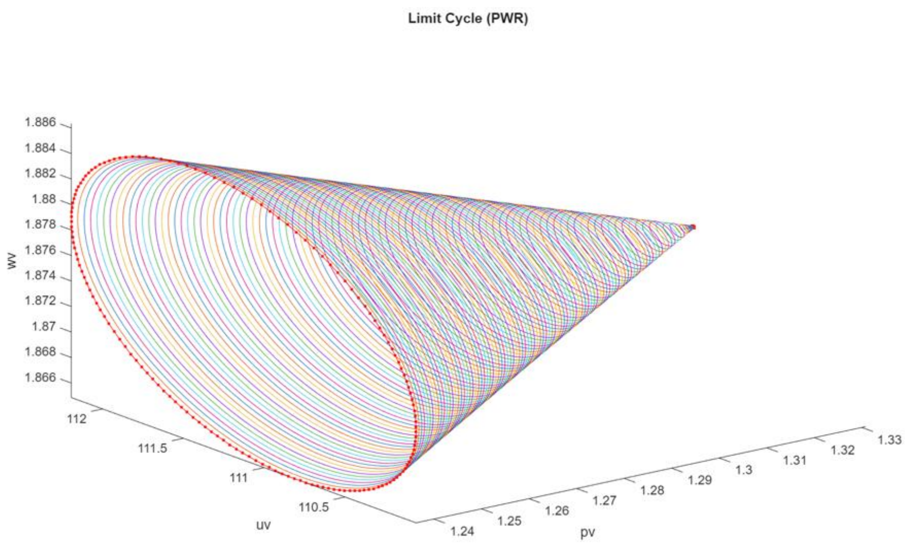

Figure. 5b: *Limit CYCLE (PWR)*

For the optimal control, $\sum_{t_{i=0}}^{t_i=t_f}(pv(t_i)+cv(t_i))$ was maximized. Initially, no measures were taken to avoid the Hopf bifurcations. The obtained value of $\sum_{t_{i=0}}^{t_i=t_f}(pv(t_i)+cv(t_i))$ was 145.6608. Subsequently, $\sum_{t_{i=0}}^{t_i=t_f}(pv(t_i)+cv(t_i))$ was minimized $(\alpha=-25)$ using the soft Hopf penalty formulation, and the obtained value of $\sum_{t_{i=0}}^{t_i=t_f}(pv(t_i)+cv(t_i))$ was 151.539. In this case, a more beneficial outcome resulted from avoiding Hopf bifurcation points. Figures 6a -6f show the optimal control profiles. In the PWR problem, the neural network used a smooth activation, a smooth tanh approximation defined by $\tanh_{approx}(x) = \frac{x}{1+|x|}$. The control profile exhibits spikes, and this was remedied using the Savitzky-Golay filter to produce the smooth control profile $\lambda_s sg$.

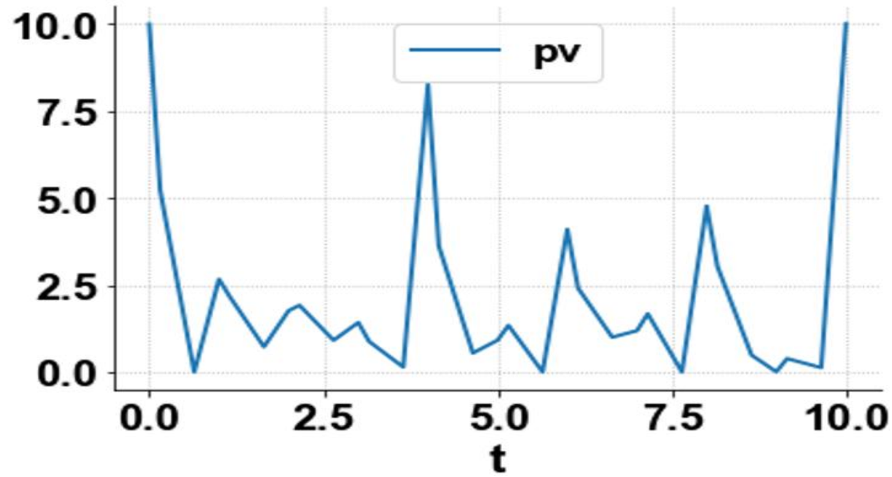

Figure 6a: *optimal control (PWR) pv profile* $(\alpha = 0)$

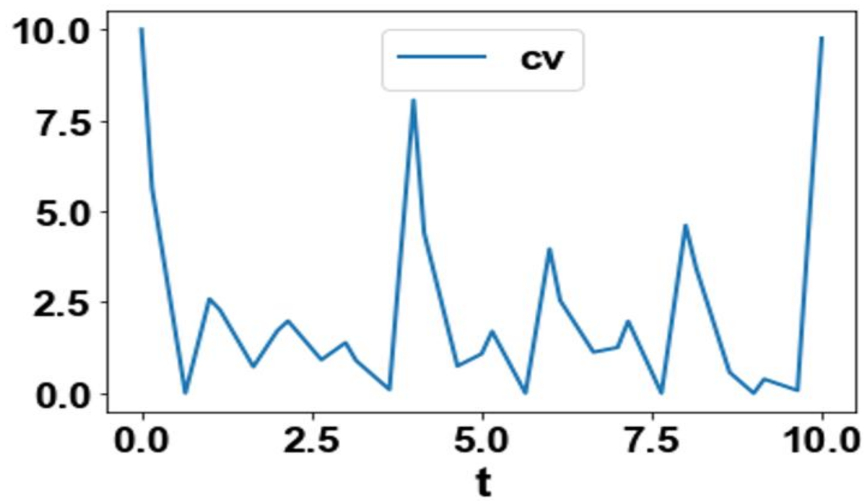

Figure 6b: *optimal control (PWR) cv profile* $(\alpha = 0)$

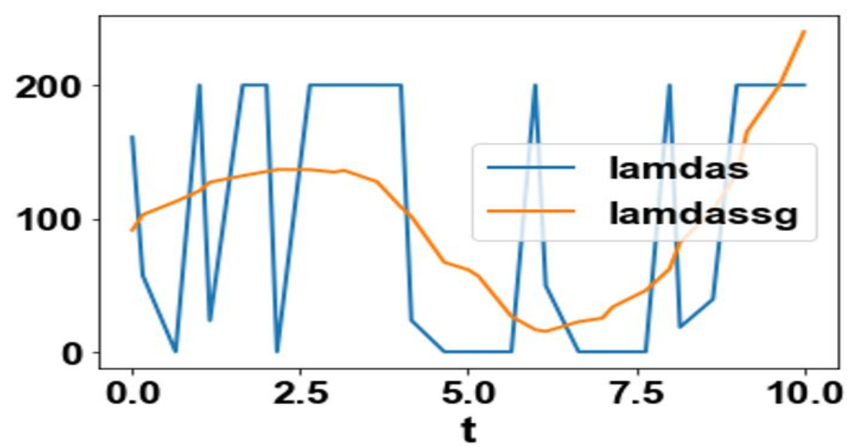

Figure 6c: *optimal control (PWR)* $\lambda_s, \lambda_s sg$ *profile* $(\alpha = 0)$

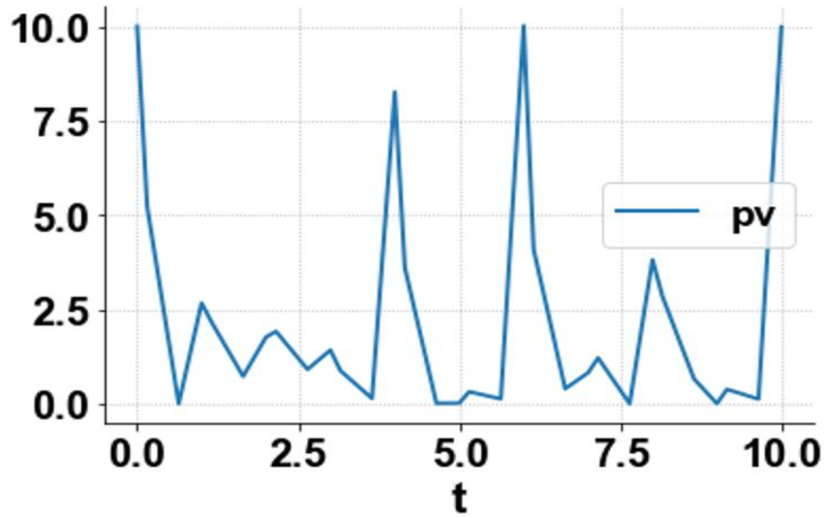

Figure 6d: *optimal control (PWR) pv profile* $(\alpha = -25)$

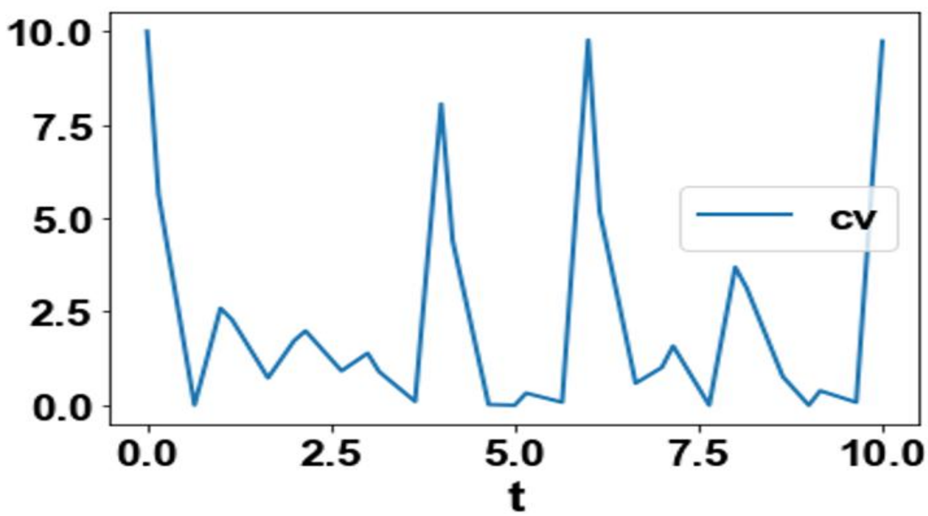

Figure 6e: *optimal control (PWR) cv profile* $(\alpha = -25)$

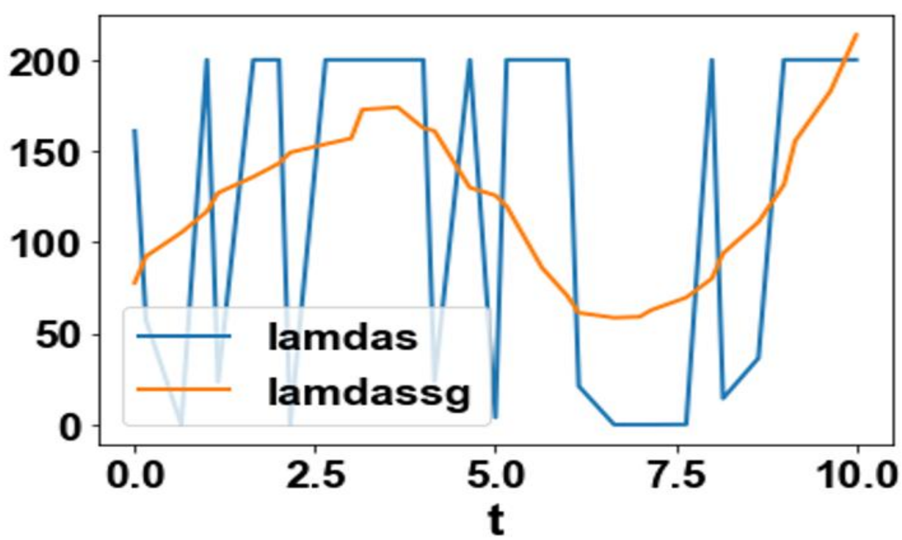

Figure 6f: *optimal control (PWR)* $\lambda_s, \lambda_s sg$ *profile* $(\alpha = -25)$

In the case of the BWR and the heavy-water nuclear reactor problems, a positive value of α produced beneficial results while avoiding the Hopf Bifurcation points. In the case of the PWR nuclear reactor problem, a negative value of α produced beneficial results while avoiding the Hopf Bifurcation points. The different behavior observed in the Hopf penalty parameter across the reactor models is related to the distinct dynamic structures and feedback mechanisms used in each model. In the BWR and heavy-water reactor models, the performance regions are near the stability boundaries, where Hopf bifurcations are possible. In these models, a positive value of the Hopf penalty parameter prevents the optimizer from entering regions with positive dominant eigenvalues. This ensures good performance without oscillations, which are associated with Hopf bifurcations.

On the other hand, in the pressurized water reactor (PWR) model, a distinct relationship between stability and performance is observed. In this case, it is possible that areas of higher objective value are located not in the unstable areas but rather in areas closer to, or even beyond, the boundaries of instability. When a negative value of the Hopf parameter is introduced, it is possible to exploit a wider range of trajectories that are closer to or even beyond the boundaries of instability. At this point, it is possible to exploit beneficial nonlinear effects in the reactor's dynamics, such as thermal-neutron coupling and feedback, to increase the value of the performance metric while avoiding regions of Hopf bifurcation.

The Hopf parameter significantly determines to what extent the optimizer is directed towards trajectories near the boundaries of stability, and its optimal value depends on the characteristics of a specific reactor model.

Conclusions:

This study investigated the application of a stability-aware optimal control framework to three nuclear reactor models: the boiling water reactor (BWR), the heavy-water reactor (HWR), and the pressurized water reactor (PWR). The objective was to integrate stability considerations associated with Hopf bifurcations directly into the dynamic optimization problem. A neural network surrogate model was used to approximate the maximum real eigenvalue of the system Jacobian, allowing the stability characteristics of the reactor dynamics to be incorporated efficiently within a Pyomo-based optimal control formulation solved using IPOPT.

The results demonstrate that the proposed approach can successfully guide the optimization process while avoiding operating regions associated with Hopf bifurcation points. For the BWR and heavy-water reactor models, using a positive Hopf penalty parameter yielded improved optimization results while maintaining stable trajectories. The penalty term effectively discouraged trajectories that approached regions where the dominant eigenvalue becomes positive, thereby preventing the onset of oscillatory behavior while still allowing the optimizer to identify favorable operating conditions.

In contrast, the pressurized water reactor model exhibited a different stability–performance relationship. In this case, negative values of the Hopf penalty parameter produced improved objective values while still avoiding Hopf bifurcation points. This result suggests that the optimal operating region of the PWR lies closer to the stability boundary and that the reactor dynamics allow beneficial operating conditions near, but not beyond, the onset of instability.

Overall, the results highlight that the interaction between performance objectives and stability boundaries can vary significantly across reactor types. The proposed neural network–based stability embedding approach provides a flexible and computationally efficient method for incorporating bifurcation avoidance into the optimal control of nuclear reactor systems. This framework offers a promising tool for identifying safe and efficient operating strategies for complex reactor dynamics.

Data Availability Statement

All data used is presented in the paper

Conflict of interest

The author, Dr. Lakshmi N Sridhar has no conflict of interest.

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