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Analysis and Control of Dengue Transmission Dynamic Models

Abstract:
In tropical areas, dengue fever is a significant public health concern. It is caused by the dengue virus, which is transmitted to humans by infected mosquitoes. Effective and efficient strategies must be implemented to minimize the damage, and to do this, we must understand the dynamics of the dengue transmission and implement control methods that are beneficial and cost-effective.

In this work, bifurcation analysis and multi objective nonlinear model predictive control is performed on two dynamic models involving dengue transmission. Bifurcation analysis is a powerful mathematical tool used to deal with the nonlinear dynamics of any process. Several factors must be considered, and multiple objectives must be met simultaneously. Bifurcation analysis and multi objective nonlinear model predictive control (MNLMPC) calculations are performed on three oncolytic dynamic models. The MATLAB program MATCONT was used to perform the bifurcation analysis. The MNLMPC calculations were performed using the optimization language PYOMO in conjunction with the state-of-the-art global optimization solvers IPOPT and BARON.The bifurcation analysis revealed the existence of branch and limit points in the models. The branch and limit points (which cause multiple steady-state solutions from a singular point) are very beneficial because they enable the Multiobjective nonlinear model predictive control calculations to converge to the Utopia point (the best possible solution) in both models.

Key Words: bifurcation; optimization; control; dengue

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Introduction:

Bhatt et al (2013)[1] discussed the global distribution and burden of dengue. Buonomo et al (2017)[2] investigated the optimal bed net use for a dengue disease model with a mosquito seasonal pattern. Agusto et al (2018)[3] discussed the optimal control strategies for dengue transmission in Pakistan. Khormi et al (2020)[4] modelled dengue fever risk based on socioeconomic parameters, meteorological factors, and vector indices in Saudi Arabia using geospatial techniques. Ndii et al (2020)[5] developed an optimal vaccination strategy for dengue transmission in Kupang city, Indonesia. Grange et al (2020)[6] discussed the evolution of endemic dengue virus in New Caledonia. Jan et al (2020)[7], developed a new model of dengue fever in terms of fractional derivatives. Abidemi et al (2020)[8] provided an analysis of dengue fever transmission dynamics with multiple controls. Abidemi et al (2020)[9] developed optimal control strategies for dengue fever spread in Johor, Malaysia. Chakraborty et al (2021)[10] analyzed the dengue transmission in bangladesh with saturated incidence rate and a constant treatment function. Khan et al (2021)[11] discussed the dynamics of dengue infection using the fractal-fractional operator with real statistical data. Hamdan et al (2021)[12] developed a deterministic dengue epidemic model with the influence of temperature. Lima-Camara et al (2021)[13], provided a review on the vectors and the epidemiology of dengue in the Americas. Asamoah et al(2021)[14] researched optimal control and cost-effectiveness analysis for dengue fever model with asymptomatic and partial immune individuals. Khan(2021)[15] modelled and analyzed dengue infection n East Java, Indonesia. Puspita et al (2022)[16] worked on the time-dependent force of infection and effective reproduction ratio in an age-structure dengue transmission model in Bandung City, Indonesia. Bonyah et al (2022)[17] developed a fractional order dengue fever model in the context of protected travelers. Aguiar et al (2022)[18] reviewed mathematical models for dengue fever epidemiology. Steindorf et al (2022)[19] modelled secondary infections with temporary immunity and disease enhancement factor and studied the mechanisms for complex dynamics in simple epidemiological models. Hamdan et al (2022)[20] modelled dengue transmission with intervention strategies using fractional derivatives. Ogunlade et al (2023)[21] provided a systematic review of mathematical models of dengue transmission and vector control. Srivastav et al (2023)[22] studied the effects of public health measures on severe dengue cases using an optimal control approach. Pongsumpun et al (2023)[23] provided a modified optimal control for the mathematical model of dengue virus with vaccination. Aldila et al (2023)[24] discussed the impact of social awareness, case detection, and hospital capacity on dengue eradication in Jakarta. Abidemi et al (2023)[25] studied the host-vector dynamics of dengue with asymptomatic, isolation and vigilant compartments with insights from modelling. Khan et al (2023)[26] investigated the dengue transmission under future climate and human population changes in mainland China. De Araújo et al (2023)[27] applied a multi-strain dengue model to epidemics data. Li et al (2023)[28] provided a dynamic analysis of an age-structured dengue model with asymptomatic infection. Vinagre et al (2023)[29] provided a dynamical analysis of a model for secondary infection of dengue. Li et al (2023)[30] performed optimal control studies of the dengue vector based on a reaction–diffusion model. Zhang et al(2023)[31] solved an optimal control problem for dengue transmission model with Wolbachia and vaccination. Barrios-Rivera (2023)[32], solved an optimal control problem of a two-patch dengue epidemic under limited resources. Saha et al (2023)[33] researched disease dynamics and optimal control strategies of a two-serotype dengue model with co-infection. Li et al (2023)[34] modelled the impact of awareness programs on the transmission dynamics of dengue and optimal control. Puspita et al (2023) [35] , modelled and analyzed dengue cases in Palu City, Indonesia. Aldila et al (2023)[36] studied the impact of social awareness, case detection,and hospital capacity on dengue eradication in Jakarta. Pongsumpun et al (2023)[37] developed a modified optimal control for the mathematical model of dengue virus with vaccination. Anam et al (2024)[38] studied within-host models unravelling the dynamics of dengue reinfections. Abidemi et al (2024)[39], developed an optimal control model for dengue dynamics with asymptomatic, isolation, and vigilant Compartments. Herdicho et al (2025)[40] modelled the dynamics of dengue Transmission with awareness and optimal control analysis.

This work aims to perform bifurcation analysis and multiobjective nonlinear control (MNLMPC) studies in two dengue transmission models, which are discussed in Abidemi et al (2024)[39] (model 1) and and . Herdicho et al (2025)[40] (model 2). The paper is organized as follows. First, the model equations are presented, followed by a discussion of the numerical techniques involving bifurcation analysis and multiobjective nonlinear model predictive control (MNLMPC). The results and discussion are then presented, followed by the conclusions.

Model Equations

Model 1(Abidemi et al (2024)[39])

In this model, *nh, sh, eh, ah, ih, qh, rh, vh nm, sm, em, im* represent the total human population, susceptible population, exposed individuals population, asymptomatic infected individual population, symptomatic infectious individual population, isolated infected individual population, recovered individual population, vigilant individual population, susceptible mosquito population, exposed mosquito population, and symptomatic infectious mosquito population. The model equations are

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$$\frac{d(sh)}{dt}=\left((1-\tau)\lambda_h\right) + \varepsilon(rh(-(\mu_h(sh)) - \frac{\left((1-\varphi_1)b(ph)sh(im)\right)}{nh}$$

$$\frac{d(eh)}{dt}=\frac{\left((1-\varphi_1)b(ph)sh(im)\right)}{nh}-\left((\sigma_h+\mu_h)eh\right)$$

$$\frac{d(ah)}{dt}=\left((1-\rho)(\sigma_h eh)\right)-\left((\sigma h1+q1+\mu_h+(\eta_1\varphi_2))ah\right);$$

$$\frac{d(ih)}{dt}=(\rho\sigma_h eh)-\left((\sigma_2+q_2+\mu_h+(\sigma_1\varphi_2) + \delta_1-(\eta_1\delta_1\varphi_2))ih\right)$$

$$\frac{d(qh)}{dt}=(q_1 ah)+(q_2 ih)-\left((\sigma 3+\mu_h+(\sigma_1\varphi_2) + \delta_2-(\eta_1\delta_2\varphi_2))qh\right)$$

$$\frac{d(rh)}{dt}=\left((1-m_1)(\sigma 1+(1\varphi 2))ah\right)+\left((1-m_2)(\sigma_2+(\eta_1\varphi_2))*ih\right)+$$
$$\left((1-m3)(\sigma_3+(\eta_1\varphi_2))qh\right)-\left((\alpha+\varepsilon+\mu_h)rh\right)$$

$$\frac{d(vh)}{dt}=(\tau\lambda_h) + \left(m_1(\sigma_1+(\eta_1\varphi_2))ah\right)+ \left(m_2(\sigma_2+(\eta_1\varphi_2))ih\right)+$$
$$\left(m3(\sigma_3+(\eta_1\varphi_2))qh\right) +(\alpha rh)-(\mu_h vh)$$

$$\frac{d(sm)}{dt}=\left((1-(\eta_2\varphi_3))\lambda_m\right) -\left((1-\varphi_1)\frac{(b(pm)sm)\left(ah+(c_1 ih)+(c_2 qh)\right)}{nh}\right)-$$
$$\left((\mu_m+(\eta_2\varphi_3))sm\right) \tag{1}$$

$$\frac{d(em)}{dt}=\left((1-\varphi_1)(b(pm)sm)\frac{\left(ah+(c_1(ih))+(c_2(qh))\right)}{nh}\right)- \left((\sigma_m+\mu_m+(\eta_2\varphi_3))em\right)$$

$$\frac{d(im)}{dt}=(\sigma_m em)-\left((\mu_m+(\eta_2\varphi_3))im\right)$$

$$nh = sh+eh+ah+ ih +qh +rh+ vh \tag{2}$$

The base parameter values are

$\lambda_h = 120.2407; \delta_1 = 0.000329; m_1 = 0.4; m_3 = 0.4; ph = 0.75; pm = 0.75; \sigma_1 = 0.328833;$
$\sigma_2 = 0.328833;\ \sigma_3 = 0.328833;\ \sigma_h = 0.12899;\ \rho = 0.5;\ q_1 = 0.19409;\ q_2 = 0.19409;$
$\tau = 0.05;\ \delta_2 = 0.000877;\ m_2 = 0.4;\ b = 0.66272;\ \varepsilon = 0.00137;\ \alpha = 0.25;\ \lambda_m = 231978.5714;$
$\mu_m = 1/42; \mu_h = 1/27010;\ c_1 = 0.75; c_2 = 0.65; \sigma_m = 0.000396; \eta_1 = 0.8; \eta_2 = 0.8;\ phi_1 = 0; \phi_2 = 0; \phi_3 = 0;$

Model 2(Herdicho et al (2025)[40])

In this model, is separated into (sm) and (im) are the susceptible and infectious mosquitoes. The unaware susceptible humans, aware susceptible humans, infectious humans, hospitalized humans and recovered humans are given by *shu, sha, ih, ph, and rh.*

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$$\frac{d(sm)}{dt} = \mu_m nm - \left(\frac{b\beta_m (sm)ih}{nh}\right) - (\mu_m sm) - (u1(\sigma)sm)$$
$$\frac{d(im)}{dt} = \left(\frac{b\beta_m (sm)ih}{nh}\right) - (\mu_m im) - (u1(\sigma)im)$$
$$\frac{d(shu)}{dt} = ((1-\tau)\lambda_h) - \left(\frac{b\beta_{hu}(shu)im}{nh}\right) - ((\mu_h + \xi)shu) - (\phi(u2)shu)$$
$$\frac{d(sha)}{dt} = (\tau\lambda_h) + (\xi(shu)) - (1-u3)\left(\frac{b\beta_{ha}(im)sha}{nh}\right) - (\mu_h(sha)) + (\phi(u2)shu)$$
$$\frac{d(ih)}{dt} = \left(\frac{b\beta_{hu}(im)shu}{nh}\right) + (1-u3)\left(\frac{b\beta_{ha}(im)sha}{nh}\right) - ((\mu_h + \varphi + \gamma)ih)$$
$$\frac{d(ph)}{dt} = (\varphi(ih)) - ((\mu_h + \varepsilon + \delta)ph)$$
$$\frac{d(rh)}{dt} = (\gamma(ih)) + (\varepsilon(ph)) - (\mu_h(rh))$$
$$nm = sm + im$$
$$nh = shu + sha + ih + ph + rh \qquad (3)$$

The base parameter values are

$\mu_m = 8.4277; \lambda_h = 46796; \mu_h = 1.172;\ b = 0.6443;\ \beta_m = 0.7445;\ \beta_{ha} = 0.4828;\ \beta_{hu} = 0.9961;$
$\delta = 0.0106;\ \varphi = 0.0702;\ \varepsilon = 0.0655;\ \gamma = 0.0614; \tau = 0.5562;\ \xi = 0.2315;$
$\sigma = 0.7;\ \phi = 0.6;\ u1 = 0.1;\ u2 = 0.001;\ u3 = 0.001;$

Bifurcation analysis

The MATLAB software MATCONT is used to perform the bifurcation calculations. Bifurcation analysis deals with multiple steady-states and limit cycles. Multiple steady states occur because of the existence of branch and limit points. Hopf bifurcation points cause limit cycles . A commonly used MATLAB program that locates limit points, branch points, and Hopf bifurcation points is MATCONT(Dhooge Govearts, and Kuznetsov, 2003[41]; Dhooge Govearts, Kuznetsov, Mestrom and Riet, 2004[42]). This program detects Limit points(LP), branch points(BP), and Hopf bifurcation points(H) for an ODE system

$$\frac{dx}{dt} = f(x, \alpha) \qquad (4)$$

$x \in R^n$ Let the bifurcation parameter be α . Since the gradient is orthogonal to the tangent vector,

The tangent plane at any point $w = [w_1, w_2, w_3, w_4, \ldots w_{n+1}]$ must satisfy

$$Aw = 0 \qquad (5)$$

Where A is

$$A = [\partial f / \partial x \quad | \partial f / \partial \alpha] \qquad (6)$$

where $\partial f / \partial x$ is the Jacobian matrix. For both limit and branch points, the Jacobian matrix $J = [\partial f / \partial x]$ must be singular. For a limit point, there is only one tangent at the point of singularity. At this singular point, there is a single non-zero vector, y, where Jy=0. This vector is of dimension n. Since there is only one tangent the vector $y = (y_1, y_2, y_3, y_4, \ldots y_n)$ must align with $\hat{w} = (w_1, w_2, w_3, w_4, \ldots w_n)$. Since

$$J\hat{w} = Aw = 0 \tag{7}$$

the n+1 th component of the tangent vector w_{n+1} = 0 at a limit point (LP).

For a branch point, there must exist two tangents at the singularity. Let the two tangents be z and w. This implies that

$$\begin{aligned} Az &= 0 \\ Aw &= 0 \end{aligned} \tag{8}$$

Consider a vector v that is orthogonal to one of the tangents (say w). v can be expressed as a linear combination of z and w ($v = \alpha z + \beta w$). Since $Az = Aw = 0$; $Av = 0$ and since w and v are orthogonal,

$w^T v = 0$. Hence $Bv = \begin{bmatrix} A \\ w^T \end{bmatrix} v = 0$ which implies that B is singular.

Hence, for a branch point (BP) the matrix $B = \begin{bmatrix} A \\ w^T \end{bmatrix}$ must be singular.

At a Hopf bifurcation point,

$$\det(2 f_x(x,\alpha) @ I_n) = 0 \tag{9}$$

@ indicates the bialternate product while I_n is the n-square identity matrix. Hopf bifurcations cause limit cycles and should be eliminated because limit cycles make optimization and control tasks very difficult. More details can be found in Kuznetsov (1998[43]; 2009[44]) and Govaerts [2000] [45].

<u>Multiobjective Nonlinear Model Predictive Control(MNLMPC)</u>

The rigorous multiobjective nonlinear model predictive control (MNLMPC) method developed by Flores Tlacuahuaz et al (2012)[46] was used.

Consider a problem where the variables $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i)$ (j=1, 2..n) have to be optimized simultaneously for a dynamic problem

$$\frac{dx}{dt} = F(x,u) \tag{10}$$

t_f being the final time value, and n the total number of objective variables and u the control parameter. The single objective optimal control problem is solved individually optimizing each of the variables $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i)$ The optimization of $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i)$ will lead to the values q_j^* . Then, the multiobjective optimal control (MOOC) problem that will be solved is

$$\begin{aligned} &\min\left(\sum_{j=1}^{n}\left(\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i) - q_j^*\right)\right)^2 \\ &subject\ to\ \frac{dx}{dt} = F(x,u); \end{aligned} \tag{11}$$

This will provide the values of u at various times. The first obtained control value of u is implemented and the rest are discarded. This procedure is repeated until the implemented and the first obtained control values are the same or if the Utopia point where ($\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i) = q_j^*$ for all j) is obtained.

Pyomo (Hart et al, 2017)[47] is used for these calculations. Here, the differential equations are converted to a Nonlinear Program (NLP) using the orthogonal collocation method The NLP is solved using IPOPT (Wächter And Biegler, 2006)[48]and confirmed as a global solution with BARON (Tawarmalani, M. and N. V. Sahinidis 2005)[49].

The steps of the algorithm are as follows

1. Optimize $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i)$ and obtain q_j^*.
2. Minimize $(\sum_{j=1}^{n}(\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i)-q_j^*))^2$ and get the control values at various times.
3. Implement the first obtained control values
4. Repeat steps 1 to 3 until there is an insignificant difference between the implemented and the first obtained value of the control variables or if the Utopia point is achieved. The Utopia point is when $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i)=q_j^*$ for all j.

Sridhar (2024)[50] demonstrated that when the bifurcation analysis revealed the presence of limit and branch points tthe MNLMPC calculations to converge to the Utopia solution . For this, the singularity condition, caused by the presence of the limit or branch points was imposed on the co-state equation (Upreti, 2013)[51]. If the minimization of q_1 lead to the value q_1^* and the minimization of q_2 lead to the value q_2^* The MNLPMC calculations will minimize the function $(q_1-q_1^*)^2+(q_2-q_2^*)^2$. The multiobjective optimal control problem is

$$\min \quad (q_1-q_1^*)^2+(q_2-q_2^*)^2 \quad subject\ to\ \frac{dx}{dt}=F(x,u) \tag{12}$$

Differentiating the objective function results in

$$\frac{d}{dx_i}((q_1-q_1^*)^2+(q_2-q_2^*)^2)=2(q_1-q_1^*)\frac{d}{dx_i}(q_1-q_1^*)+2(q_2-q_2^*)\frac{d}{dx_i}(q_2-q_2^*) \tag{13}$$

The Utopia point requires that both $(q_1-q_1^*)$ and $(q_2-q_2^*)$ are zero. Hence

$$\frac{d}{dx_i}((q_1-q_1^*)^2+(q_2-q_2^*)^2)=0 \tag{14}$$

The optimal control co-state equation (Upreti; 2013)[51] is

$$\frac{d}{dt}(\lambda_i)=-\frac{d}{dx_i}((q_1-q_1^*)^2+(q_2-q_2^*)^2)-f_x\lambda_i; \quad \lambda_i(t_f)=0 \tag{15}$$

λ_i is the Lagrangian multiplier. t_f is the final time. The first term in this equation is 0 and hence

$$\frac{d}{dt}(\lambda_i)=-f_x\lambda_i;\lambda_i(t_f)=0 \tag{16}$$

At a limit or a branch point, for the set of ODE $\frac{dx}{dt}=f(x,u)$ f_x is singular. Hence there are two different vectors-values for

$[\lambda_i]$ where $\frac{d}{dt}(\lambda_i) > 0$ and $\frac{d}{dt}(\lambda_i) < 0$. In between there is a vector $[\lambda_i]$ where $\frac{d}{dt}(\lambda_i) = 0$. This coupled with the boundary condition $\lambda_i(t_f) = 0$ will lead to $[\lambda_i] = 0$ This makes the problem an unconstrained optimization problem, and the optimal solution is the Utopia solution.

Results and Discussion

The bifurcation analysis of the Dengue model 1 revealed a branch point at [sh, eh, ah, ih, qh, rh, vh, sm, em,im φ_1] values of
(3085316.24165, 0, 0, 0, 0, 0, 162385.06535, 9743099.9988, 0, 0, 0.072616). This is shown in Fig. 1.
For theMNLMPC calculations in model 1, sh(0) eh(0) ih(0) qh(0) ah(0) rh(0) vh(0) sm(0) em(0) im(0) values are 3247302, 120, 85, 37, 61, 45, 50, 100,100.

$\sum_{t_{i=0}}^{t_i=t_f} ah(t_i), \sum_{t_{i=0}}^{t_i=t_f} ih(t_i), \sum_{t_{i=0}}^{t_i=t_f} qh(t_i), \sum_{t_{i=0}}^{t_i=t_f} em(t_i), \sum_{t_{i=0}}^{t_i=t_f} im(t_i)$ were minimized individually and each of them led to a values 85, 37, 61, 100, and 100. The overall optimal control problem will involve the minimization of

$$(\sum_{t_{i=0}}^{t_i=t_f} ah(t_i) - 85)^2 (\sum_{t_{i=0}}^{t_i=t_f} ih(t_i) - 37)^2 + (\sum_{t_{i=0}}^{t_i=t_f} qh(t_i) - 61)^2 + (\sum_{t_{i=0}}^{t_i=t_f} em(t_i) - 100)^2 + (\sum_{t_{i=0}}^{t_i=t_f} im(t_i) - 100)^2$$

was minimized subject to the equations governing the model. This led to a value of zero (the Utopia solution. The various concentration profiles for this MNLMPC calculation are shown in Figs. 2-13.

The MNLMPC values of the control variables, ϕ_1, ϕ_2, ϕ_3 were 0.18911, 0.07406, and 0.009789.

The control profiles, ϕ_1, ϕ_2, ϕ_3 (Fig. 12) exhibited noise, and this was remedied using the Savitzky Golay filter (Fig. 13). It is seen that the presence of the branch point is beneficial because it allows the MNLMPC calculations to attain the Utopia solution, validating the analysis of Sridhar(2024).
The bifurcation analysis of the Dengue model 2 (u1 was the bifurcation parameter) revealed a limit point at (sm; im; shu; sha; ih; ph; rh u1) ; values of (1366836.962844 0, 14791.015455, 25137.31219, 0, 0, 0, 0)

For theMNLMPC calculations in model 2, sm(0) shu(0) sha(0) were 5.0e+05, 1.0e+05 and 1.0e+05.

$\sum_{t_{i=0}}^{t_i=t_f} im(t_i), \sum_{t_{i=0}}^{t_i=t_f} ih(t_i), \sum_{t_{i=0}}^{t_i=t_f} ph(t_i)$ were minimized individually and each of them led to a values 0. The overall optimal 5, 37, 61, 100, and 100. control problem will involve the minimization of $(\sum_{t_{i=0}}^{t_i=t_f} im(t_i))^2 (\sum_{t_{i=0}}^{t_i=t_f} ih(t_i))^2 + (\sum_{t_{i=0}}^{t_i=t_f} ph(t_i))^2$

was minimized subject to the equations governing the model. This led to a value of zero (the Utopia
The MNLMPC values of the control variables, u1, u2 u3 were (0.01656, 0.47262, 0.465221)The control profiles , u1, u2, u3 ϕ_1, ϕ_2, ϕ_3 (Fig. 22) exhibited noise, and this was remedied using the Savitzky-Golay filter (Fig. 23). It is seen that the presence of the limit point is beneficial because it allows the MNLMPC calculations to attain the Utopia solution, validating the analysis of Sridhar(2024)[50].

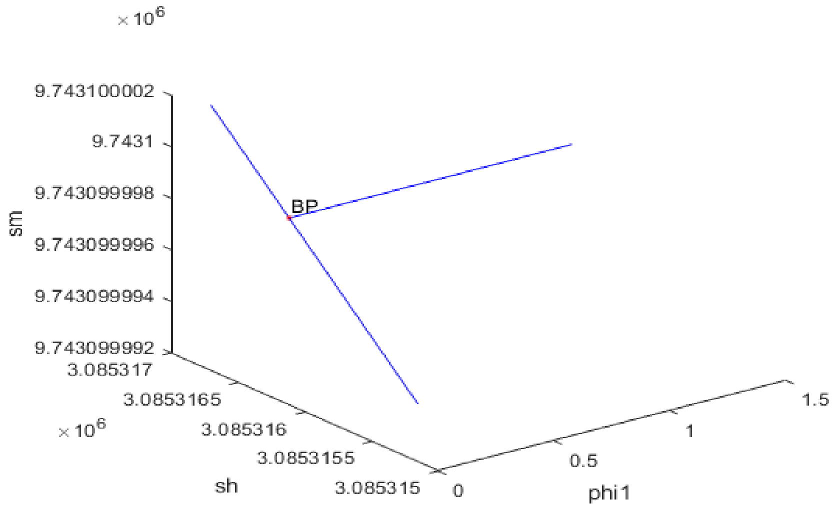

Figure 1: *Bifurcation Analysis for Dengue Model 1*

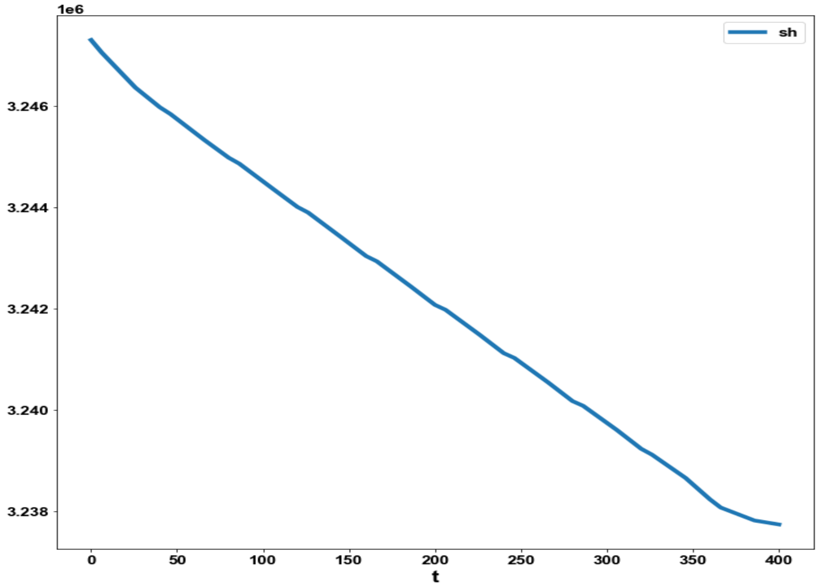

Figure 2: *MNLMPC Dengue Model1 sh vs t*

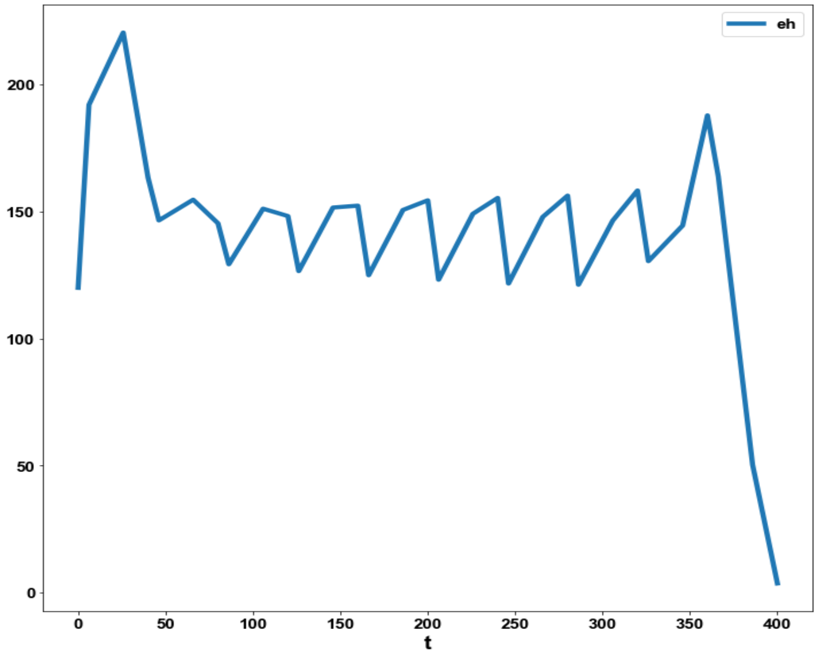

Figure 3 : *MNLMPC Dengue Model 1 eh vs t*

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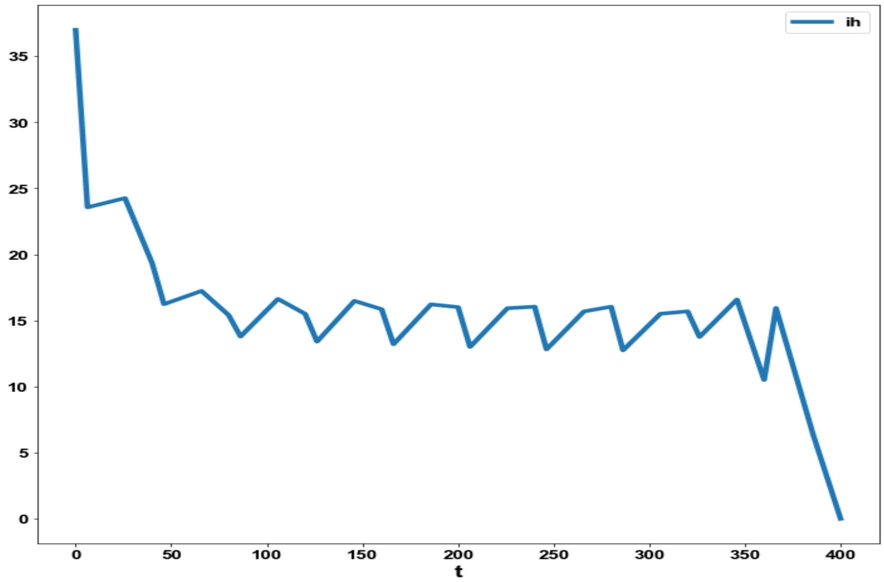

Figure 4 : *MNLMPC Dengue Model1 ih vs t*

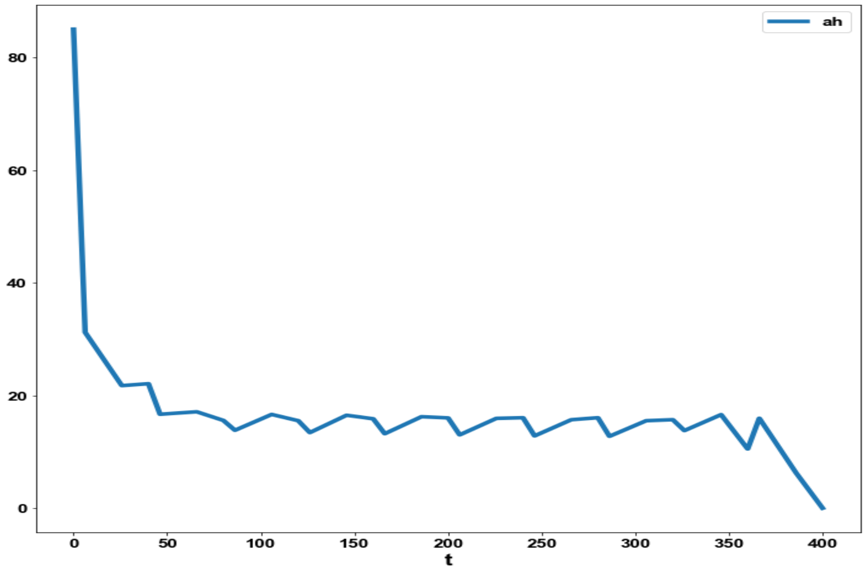

Figure 5 : *MNLMPC Dengue Model 1 ah vs t*

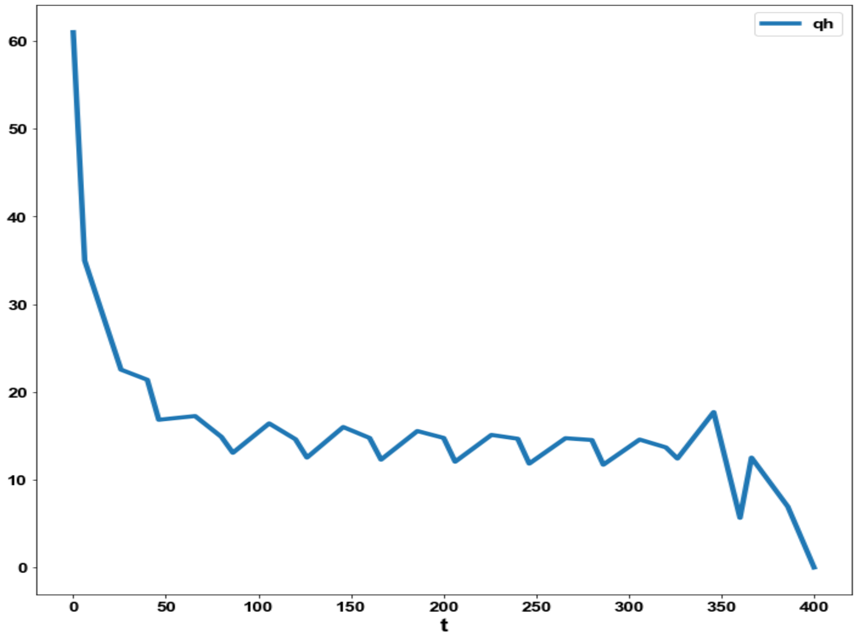

Figure 6: *MNLMPC Dengue Model1 qh vs t*

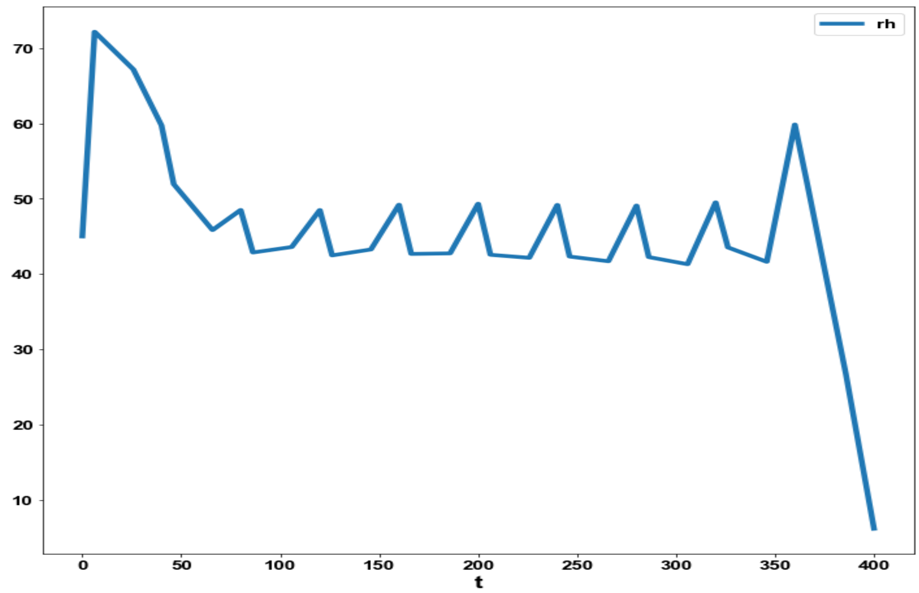

Figure 7: *MNLMPC Dengue Model 1 rh vs t*

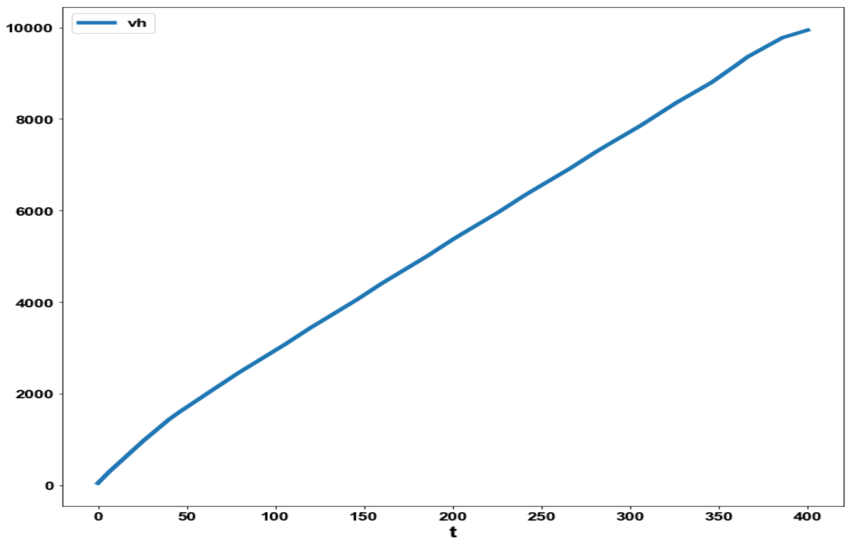

Figure 8: *MNLMPC Dengue Model 1 vh vs t*

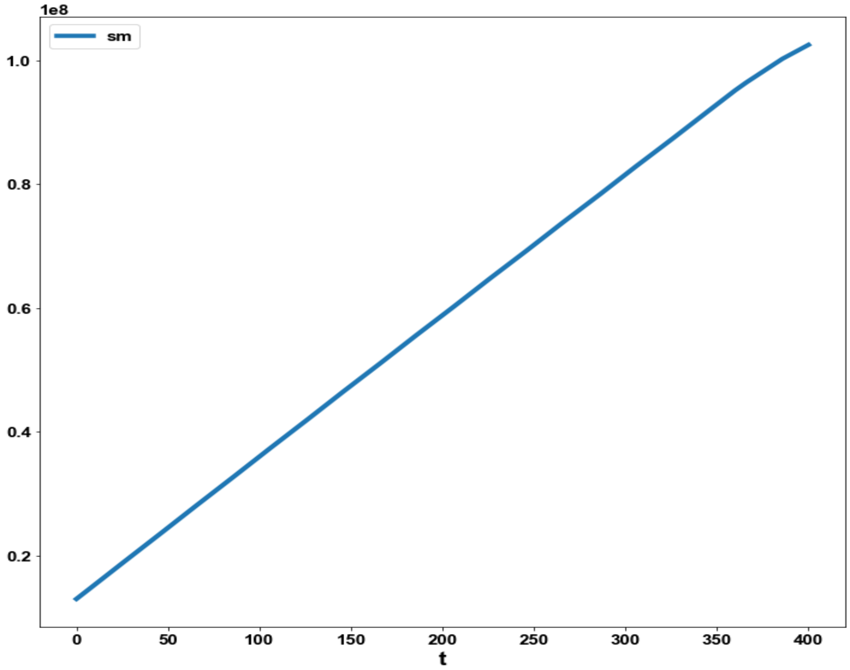

Figure 9: *MNLMPC Dengue Model 1 sm vs t*

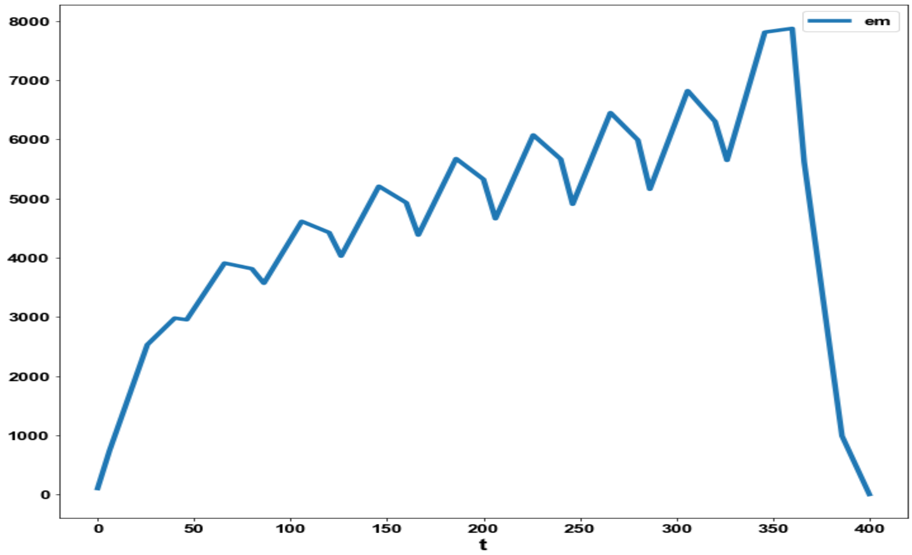

Figure 10 *: MNLMPC Dengue Model 1 em vs t*

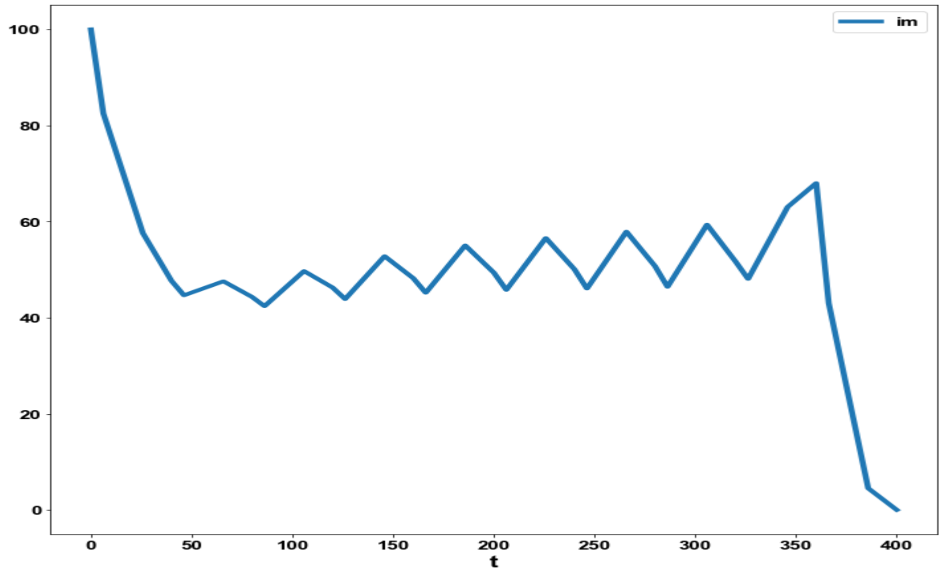

Figure 11: *MNLMPC Dengue Model 1 im vs t*

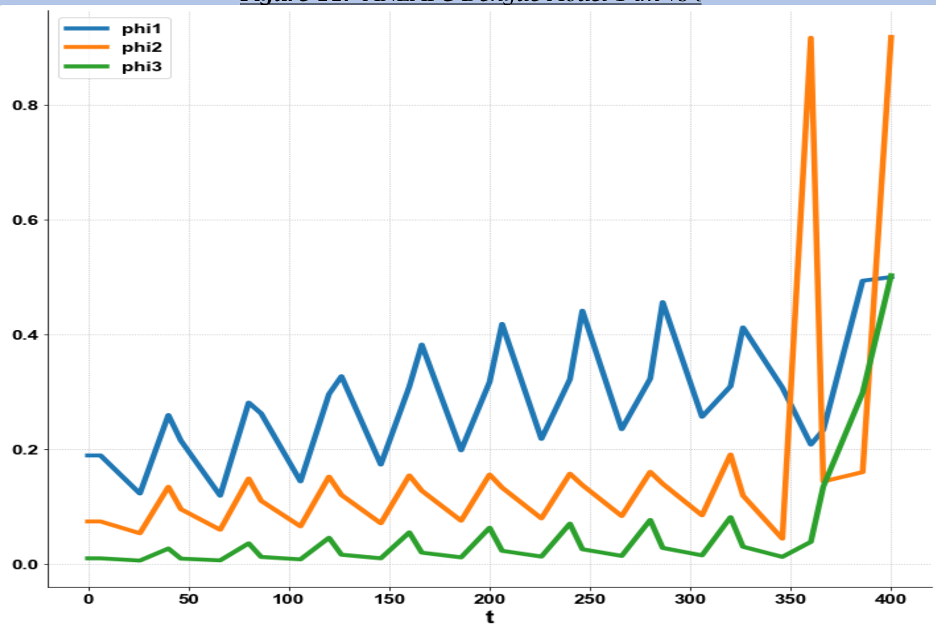

Figure 12: *MNLMPC Dengue Model 1 phi1 phi2 phi3 vs t (noise exhibited)*

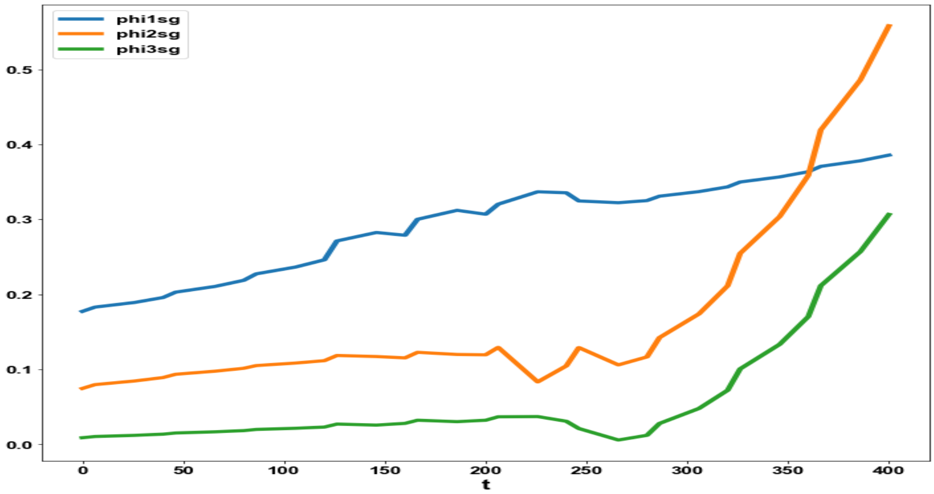

Figure 13: *MNLMPC Dengue Model 1 phi1sg phi2sg phi3sg vs t (noise removed with Savitzky-Golay filter)*

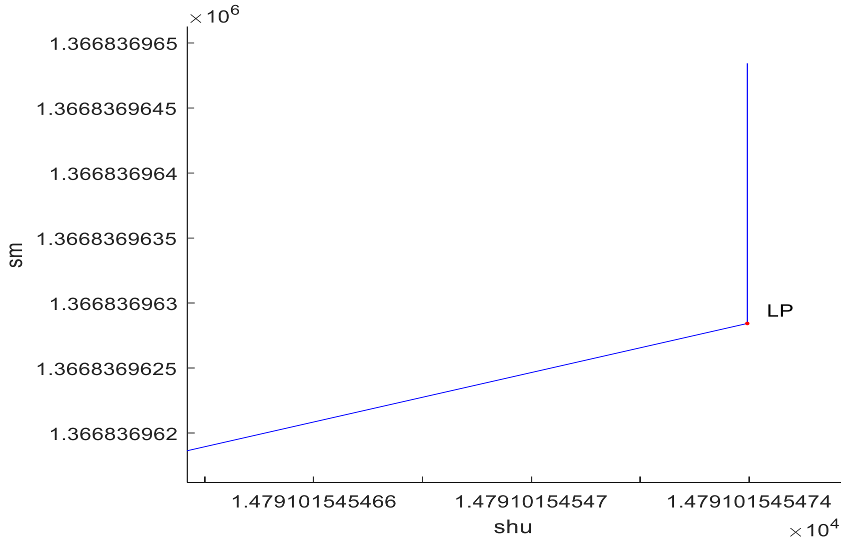

Figure 14: *Bifurcation Analysis for Dengue Model 2*

Conclusions:

Bifurcation analysis and multiobjective nonlinear control (MNLMPC) studies in two dynamic dengue transmission models. The bifurcation analysis revealed the existence of branch and limit points. The branch and limit points (which cause multiple steady-state solutions from a singular point) are very beneficial because they enable the Multiobjective nonlinear model predictive control calculations to converge to the Utopia point (the best possible solution) in the models. A combination of bifurcation analysis and Multiobjective Nonlinear Model Predictive Control(MNLMPC) for dengue transmission models is the main contribution of this paper.

Data Availability Statement

All data used is presented in the paper

Conflict of interest

The author, Dr. Lakshmi N Sridhar has no conflict of interest.

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