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Analysis and control of a Dynamic Model Involving Students Anxiety towards Mathematics

Abstract:

Anxiety towards mathematics is the most common problem for many students in several universities. In this study, bifurcation analysis and multiobjective nonlinear model predictive control (MNLMPC) calculations are performed on a dynamic model involving students' anxiety towards mathematics. Bifurcation analysis is a powerful mathematical tool used to deal with the nonlinear dynamics of any process. Several factors must be considered, and multiple objectives must be met simultaneously. The MATLAB program MATCONT was used to perform the bifurcation analysis. The MNLMPC calculations were performed using the optimization language PYOMO in conjunction with the state-of-the-art global optimization solvers IPOPT and BARON.The bifurcation analysis reveals a branch point and a Hopf bifurcation point, which leads to a limit cycle. This Hopf point was eliminated using an activation factor that involves the tanh function. The branch point enables the multiobjective nonlinear model predictive control calculations to converge to the Utopia point, which is the best solution.

Key Words: bifurcation; optimization; control; mathematics; hopf

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Introduction:

Mohamed, and Tarmizi(2010)[1], investigated anxiety in mathematics learning among secondary school learners making a comparative study between Tanzania and Malaysia. Akin and Kurbanoglu(2011)[2] studied the relationships between math anxiety, math attitudes, and self-efficacy using a structural equation model. Maria de Lourdes et al (2012) [3] conducted research on the effects of individual, motivational, and social support factors on attitudes towards mathematics. Hoorfar and, Taleb (2015) [4] developed a correlation between mathematics anxiety with metacognitive knowledge. Getahun et al (2016) [5] , discussed strategies to predict mathematics performance considering anxiety, enjoyment, value, and self-efficacy beliefs towards mathematics among engineering majors. Zakaria et al (2017) [6] studied the effect of a realistic mathematics education approach on students' achievement and attitudes towards mathematics. Mazana et al (2019) [7] investigated students' attitudes towards learning mathematics. Mazana et al (2020) [8] provided a teacher's perspective regarding assessing students' performance in mathematics in Tanzania. Teklu et al (2022) [9], provided a mathematical modeling analysis on the dynamics of university students animosity towards mathematics using optimal control. Teklu (2023) [10] developed an analysis of a fractional order model on higher institution students' anxiety towards mathematics with optimal control. This work aims to perform bifurcation analysis and multiobjective nonlinear control (MNLMPC) studies on a dynamic model concerning students' anxiety towards mathematics in Teklu (2023) [10]. The paper is organized as follows. First, the model equations are presented, followed by a discussion of the numerical techniques involving bifurcation analysis and multiobjective nonlinear model predictive control (MNLMPC). The results and discussion are then presented, followed by the conclusions.

1. Model Equations(Teklu (2023)[10]

The model equations are

$$\begin{aligned}
\frac{d(sh)}{dt} &= \left((1-\varepsilon)\Delta\right) + \omega(rh) + \rho(1-k)ph - \left(\left((1-c1)\lambda_n\right) + \mu\right)sh \\
\frac{d(ph)}{dt} &= \varepsilon\Delta - \left(\mu + (1-k)\rho\right)ph \\
\frac{d(eh)}{dt} &= \left((1-c1)\lambda_n\right)sh - (\mu+\psi)eh \\
\frac{d(ah)}{dt} &= (1-\sigma)\psi(eh) - \left(\mu+\delta+(c2\gamma)\right)ah \\
\frac{d(qh)}{dt} &= \left(\delta(ah)\right) - \left(\mu(qh)\right) \\
\frac{d(rh)}{dt} &= \left(c2(\gamma)\right)ah + \left(\sigma\psi(eh)\right) - (\mu+\omega)rh \\
nh &= sh + ph + eh + ah + qh + rh \\
\lambda_n &= \frac{\beta}{nh}\left(ah + \left(\varphi(qh)\right)\right)
\end{aligned} \tag{1}$$

The variables *sh,ph,eh,ah,qh,* and *rh* represent anxiety towards mathematics susceptible students, anxiety towards mathematics protected students, anxiety towards mathematics exposed students, students who have anxiety towards mathematics, students who have permanent anxiety towards mathematics, students recovered from anxiety towards mathematics

The base parameters are

$$\Delta = 100; \mu = 0.5; \psi = 0.04; \varepsilon = 0.4; k = 0.8; \gamma = 0.01; \omega = 0.03; \rho = 0.2;$$
$$\delta = 0.3; \sigma = 0.04; \beta = 1.4; c1 = 0; \varphi = 1.3; c2 = 0$$

Bifurcation analysis :

The MATLAB software MATCONT is used to perform the bifurcation calculations. Bifurcation analysis deals with multiple steady-states and limit cycles. Multiple steady states occur because of the existence of branch and limit points. Hopf bifurcation points cause limit cycles . A commonly used MATLAB program that locates limit points, branch points, and Hopf bifurcation points is MATCONT(Dhooge Govearts, and Kuznetsov, 2003[11]; Dhooge Govearts, Kuznetsov, Mestrom and Riet, 2004[12]). This program detects Limit points (LP), branch points (BP), and Hopf bifurcation points(H) for an ODE system

$$\frac{dx}{dt} = f(x,\alpha) \tag{2}$$

$x \in R^n$ Let the bifurcation parameter be α . Since the gradient is orthogonal to the tangent vector,

The tangent plane at any point $z = [z_1, z_2, z_3, z_4, \ldots z_{n+1}]$ must satisfy

$$Az = 0 \quad (3)$$

Where A is

$$A = [\partial f / \partial x \quad | \partial f / \partial \alpha] \quad (4)$$

where $\partial f / \partial x$ is the Jacobian matrix. For both limit and branch points, the matrix $[\partial f / \partial x]$ must be singular. The n+1 th component of the tangent vector Z_{n+1} = 0 for a limit point (LP)and for a branch point (BP) the matrix $\begin{bmatrix} A \\ z^T \end{bmatrix}$ must be singular. At a Hopf bifurcation point,

$$\det(2 f_x(x,\alpha) @ I_n) = 0 \quad (5)$$

@ indicates the bialternate product while I_n is the n-square identity matrix. Hopf bifurcations cause limit cycles and should be eliminated because limit cycles make optimization and control tasks very difficult. More details can be found in Kuznetsov (1998 [13]; 2009[14]) and Govaerts [2000] [15].

Hopf bifurcations cause unwanted oscillatory behavior and limit cycles. The tanh activation function (where a control value u is replaced by) $(u \tanh u / \varepsilon)$ is commonly used in neural nets (Dubey et al 2022[16]; Kamalov et al, 2021[17] and Szandała, 2020[18]) and optimal control problems(Sridhar 2023[19]) to eliminate spikes in the optimal control profile. Hopf bifurcation points cause oscillatory behavior. Oscillations are similar to spikes, and the results in Sridhar(2024)[20] demonstrate that the tanh factor also eliminates the Hopf bifurcation by preventing the occurrence of oscillations. Sridhar (2024)[20] explained with several examples how the activation factor involving the tanh function successfully eliminates the limit cycle causing Hopf bifurcation points. This was because the tanh function increases the time period of the oscillatory behavior, which occurs in the form of a limit cycle caused by Hopf bifurcations.

Multiobjective Nonlinear Model Predictive Control (MNLMPC)

Flores Tlacuahuaz et al (2012)[21] developed a multiobjective nonlinear model predictive control (MNLMPC) method that is rigorous and does not involve weighting functions or additional constraints. This procedure is used for performing the MNLMPC calculations Here $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i)$ (j=1, 2..n) represents the variables that need to be minimized/maximized simultaneously for a problem involving a set of ODE

$$\frac{dx}{dt} = F(x,u) \quad (6)$$

t_f being the final time value, and n the total number of objective variables and . u the control parameter. This MNLMPC procedure first solves the single objective optimal control problem independently optimizing each of the variables $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i)$ individually. The minimization/maximization of $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i)$ will lead to the values q_j^* . Then the optimization problem that will be solved is

$$\min\left(\sum_{j=1}^{n}\left(\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i) - q_j^*\right)\right)^2$$
$$subject\ to\ \frac{dx}{dt} = F(x,u); \quad (7)$$

This will provide the values of u at various times. The first obtained control value of u is implemented and the rest are discarded. This procedure is repeated until the implemented and the first obtained control values are the same or if the Utopia point where (

$\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i) = q_j^*$ for all j) is obtained.

Pyomo (Hart et al, 2017)[22] is used for these calculations. Here, the differential equations are converted to a Nonlinear Program (NLP) using the orthogonal collocation method The NLP is solved using IPOPT (Wächter And Biegler, 2006)[23]and confirmed as a global solution with BARON (Tawarmalani, M. and N. V. Sahinidis 2005)[24].

The steps of the algorithm are as follows

1. Optimize $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i)$ and obtain q_j^* at various time intervals t_i. The subscript i is the index for each time step.

2. Minimize $(\sum_{j=1}^{n}(\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i) - q_j^*))^2$ and get the control values for various times.

3. Implement the first obtained control values

4. Repeat steps 1 to 3 until there is an insignificant difference between the implemented and the first obtained value of the control variables or if the Utopia point is achieved. The Utopia point is when $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i) = q_j^*$ for all j.

Sridhar (2024)[25] proved that the MNLMPC calculations to converge to the Utopia solution when the bifurcation analysis revealed the presence of limit and branch points . This was done by imposing the singularity condition on the co-state equation (Upreti, 2013)[26]. If the minimization of q_1 lead to the value q_1^* and the minimization of q_2 lead to the value q_2^* The MNLPMC calculations will minimize the function $(q_1 - q_1^*)^2 + (q_2 - q_2^*)^2$. The multiobjective optimal control problem is

$$\min \quad (q_1 - q_1^*)^2 + (q_2 - q_2^*)^2 \quad subject\ to \quad \frac{dx}{dt} = F(x,u) \tag{8}$$

Differentiating the objective function results in

$$\frac{d}{dx_i}((q_1 - q_1^*)^2 + (q_2 - q_2^*)^2) = 2(q_1 - q_1^*)\frac{d}{dx_i}(q_1 - q_1^*) + 2(q_2 - q_2^*)\frac{d}{dx_i}(q_2 - q_2^*) \tag{9}$$

The Utopia point requires that both $(q_1 - q_1^*)$ and $(q_2 - q_2^*)$ are zero. Hence

$$\frac{d}{dx_i}((q_1 - q_1^*)^2 + (q_2 - q_2^*)^2) = 0 \tag{10}$$

the optimal control co-state equation (Upreti; 2013)[53] is

$$\frac{d}{dt}(\lambda_i) = -\frac{d}{dx_i}((q_1 - q_1^*)^2 + (q_2 - q_2^*)^2) - f_x\lambda_i; \quad \lambda_i(t_f) = 0 \tag{11}$$

λ_i is the Lagrangian multiplier. t_f is the final time. The first term in this equation is 0 and hence

$$\frac{d}{dt}(\lambda_i) = -f_x\lambda_i; \lambda_i(t_f) = 0 \quad (12)$$

At a limit or a branch point, for the set of ODEs $\frac{dx}{dt} = f(x,u)$ f_x is singular. Hence, there are two different vector values for $[\lambda_i]$ where $\frac{d}{dt}(\lambda_i) > 0$ and $\frac{d}{dt}(\lambda_i) < 0$. In between there is a vector $[\lambda_i]$ where $\frac{d}{dt}(\lambda_i) = 0$. This, coupled with the boundary condition $\lambda_i(t_f) = 0$ will lead to $[\lambda_i] = 0$ This makes the problem an unconstrained optimization problem, and the only solution is the Utopia solution.

Results and Discussion :

For the bifurcation analysis ψ used as the bifurcation parameter. A Hopf bifurcation point and a branch point were located at $[sh, ph, eh, ah, qh, rh, \psi]$ values of (125.925926, 74.074074, 0, 0, 0, 0, 0.943090) and (125.925926, 74.074074, 0, 0, 0, 0,0.566351) as shown in Fig. 1. Fig. 2 shows the limit cycle that occurs because of the Hopf bifurcation. When the tanh activation factor is used and ψ is modified to $\psi\tanh(\psi)$ the Hopf bifurcation point disappears, and 2 branch points are found at $[sh, ph, eh, ah, qh, rh, \psi]$ values of
(125.925926 74.074074 0, 0, 0, 0, -0.831381) and (125.925926 74.074074 0, 0, 0, 0, 0.831381). This is shown in Fig. 3. The use of the tanh activation function successfully eliminates the Hopf bifurcation that causes a limit cycle, thereby validating the analysis by Sridhar (2024)[20].

For the MNLMPC calculations, $\sum_{t_{i=0}}^{t_i=t_f} ah_j(t_i), \sum_{t_{i=0}}^{t_i=t_f} eh_j(t_i), \sum_{t_{i=0}}^{t_i=t_f} c1_j(t_i), \sum_{t_{i=0}}^{t_i=t_f} c2_j(t_i)$ were minimized, and each minimization resulted in a value of 0. c1 and c2 were used as the control parameters. The multiobjective optimal control calculation involved a minimization of $(\sum_{t_{i=0}}^{t_i=t_f} ah_j(t_i)-0)^2 + (\sum_{t_{i=0}}^{t_i=t_f} eh_j(t_i)-0)^2 + (\sum_{t_{i=0}}^{t_i=t_f} c1_j(t_i)-0)^2 + (\sum_{t_{i=0}}^{t_i=t_f} c2_j(t_i)-0)^2$ and resulted in the Utopia solution, validating the analysis in Sridhar(2024)[25]. The MNLMPC values of c1 and c2 were 0.16365 and 0.49954. Figures 4-9 show the various MNLMPC profiles. In Fig. 9 it is seen that the c1 vs t profile exhibits noise, which is removed using the Savitzky-Golay filter to produce a smooth profile (c1sg). The use of the tanh function eliminates the Hopf bifurcation point, validating the analysis in Sridhar (2024)[20], and the convergence of the MNLMPC calculations to the Utopia solution validates the analysis in Sridhar (2024)[25].

Conclusions

Multiobjective nonlinear model predictive control calculations were performed along with bifurcation analysis on a model involving students' anxiety towards Mathematics. The bifurcation analysis revealed the existence of a limit cycle causing Hopf bifurcation point, and a branch point. The Hopf bifurcation point is eliminated using an activation factor involving the tanh function. The presence of the branch-point enables the multiojective nonlinear model predictive calculations to converge to the Utopia point which is the best possible solution.

Data Availability Statement:

All data used is presented in the paper

Conflict of interest

The author, Dr. Lakshmi N Sridhar has no conflict of interest.

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