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Analysis and Control of the Activated Sludge Model (ASM1)

Abstract:
Elimination of contamination in wastewater is crucial to ensure the well-being and health of the population. The activated sludge process is highly nonlinear, and many factors must be taken into account to ensure that the process is conducted most efficiently. Bifurcation analysis is a powerful mathematical tool used to deal with the nonlinear dynamics of any process. Several factors must be considered, and multiple objectives must be met simultaneously. Bifurcation analysis and multiobjective nonlinear model predictive control (MNLMPC) calculations are performed on the activated sludge model (ASM1). The MATLAB program MATCONT was used to perform the bifurcation analysis. The MNLMPC calculations were performed using the optimization language PYOMO in conjunction with the state-of-the-art global optimization solvers IPOPT and BARON. The bifurcation analysis revealed the existence of branch points in the model. The branch points were beneficial because they enabled the multiobjective nonlinear model predictive control calculations to converge to the Utopia point in both problems, which is the most beneficial solution. A combination of bifurcation analysis and multiobjective nonlinear model predictive control for the activated sludge model (ASM1) is the main contribution of this paper.

Key Words: activated sludge model (ASM1); bifurcation; optimization; control

Author Information

Lakshmi. N. Sridhar*
Chemical Engineering Department University of Puerto Rico Mayaguez

***Corresponding Author:** Lakshmi. N. Sridhar, Chemical Engineering Department University of Puerto Rico Mayaguez

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Introduction:

To minimize effluent contamination concentrations, wastewater treatment plants use the activated sludge process. This process should be conducted efficiently, keeping all unnecessary expenses to a minimum. To achieve this goal, there has been a lot of modelling work to understand the various chemical reactions involved in this process. Henze et al (1987) [1] developed a general model for single-sludge wastewater treatment systems. Henze et al (1995) [2] extended and improved this earlier model.

Henze (1999) [3] performed modelling work on the aerobic wastewater treatment processes taking into account environmental impacts. Gujer et al (1995) [4] further improved upon the models of Henze. Fikar et al (2005) [5] developed strategies to ensure the optimal operation of alternating activated sludge processes. Yoon et al (2005) [6], Critical operational parameters for zero sludge production in biological wastewater treatment processes combined with sludge disintegration

Nelson et al (2009) [7] used continuation methods to determine the steady-state behaviour of the activated sludge model (ASM1).

The activated sludge models are highly nonlinear, and many factors must be taken into account to ensure that the process is conducted most efficiently. In this article, a combination of bifurcation analysis and multi objective nonlinear model predictive control (MNLMPC) for the activated sludge model (ASM1) (Nelson et al, 2009) [7] is performed. The bifurcation analysis reveals the presence of branch points, which are very beneficial because they enable the MNLMPC calculations to converge to the Utopia point, which is the best possible solution. This paper is organized as follows. First, the ASM1 model equations) (Nelson et al, 2009) [7] are presented. The numerical procedures (bifurcation analysis and multiobjective nonlinear model predictive control (MNLMPC) are then described. This is followed by the results and discussion and conclusions.

ASM1 model equations:

$$\frac{dS_S}{dt} = d(S_{S,in} - S_S) - \frac{\mu_{MAX,H}}{Y_H} M_2 (M_{8h} + I_8 M_9 \eta_g) X_{Bh} + k_h k_{sat} (M_{8h} + I_8 M_9 \eta_h) X_{Bh}$$
$$\frac{dX_S}{dt} = d(X_{S,in} - X_S) + d(b-1) X_S + (1 - f_p)(b_H X_{BA} + b_A X_{BA}) - k_h k_{sat} (M_{8h} + I_8 M_9 \eta_h) X_{Bh}$$
$$\frac{dX_{BH}}{dt} = d(X_{BH,in} - X_{BH}) + d(b-1) X_{BH} + \mu_{MAX,H} M_2 (M_{8h} + I_8 M_9 \eta_g) X_{BH} - b_H X_{BH}$$
$$\frac{dX_{BA}}{dt} = d(X_{BA,in} - X_{BA}) + d(b-1) X_{BA} + \mu_{MAX,A} M_{10} M_{8A} X_{BA} - b_H X_{BA}$$
$$\frac{dS_O}{dt} = d(S_{O,in} - S_O) - \frac{(1-Y_H)}{Y_H} \mu_{MAX,H} M_2 M_{8h} X_{BH} - \frac{(4.57 - Y_A)}{Y_A} \mu_{MAX,A} M_{10} M_{8A} X_{BA}$$
$$\frac{dS_{NO}}{dt} = d(S_{NO,in} - S_{NO}) - \frac{(1-Y_H)}{2.86 Y_H} \mu_{MAX,H} M_2 I_8 M_9 \eta_g X_{BH} + \frac{1}{Y_A} \mu_{MAX,A} M_{10} M_{8A} X_{BA}$$
$$\frac{dS_{NH}}{dt} = d(S_{NH,in} - S_{NH}) - i_{XB} \mu_{MAX,H} M_2 (M_{8h} + I_8 M_9 \eta_g) X_{BH} - (i_{XB} + \frac{1}{Y_A}) \mu_{MAX,A} M_{10} M_{8A} X_{BA} + K_A S_{ND} X_{BH}$$
$$\frac{dS_{ND}}{dt} = d(S_{ND,in} - S_{ND}) - K_A S_{ND} X_{BH} + K_H K_{SAT} (M_{8h} + I_8 M_9 \eta_g) X_{BH} \frac{X_{ND}}{X_S}$$
$$\frac{dX_{ND}}{dt} = d(X_{ND,in} - X_{ND}) + +d(b-1) X_{ND} + (i_{XB} - f_p i_{XP})(b_H X_{BA} + b_A X_{BA}) - K_H K_{SAT} (M_{8h} + I_8 M_9 \eta_g) X_{BH} \frac{X_{ND}}{X_S} \quad (1)$$

Were

$$M_2 = \frac{S_s}{(K_s + S_s)}; M_{8a} = \frac{S_o}{(K_{oa} + S_o)}; M_{8h} = \frac{S_o}{(K_{oh} + S_o)}; M_9 = \frac{S_{No}}{(K_{No} + S_{No})};$$
$$M_{10} = \frac{S_{NH}}{(K_{NH} + S_{NH})}; I_8 = \frac{K_{oh}}{(K_{oh} + S_o)}; K_{sat} = \frac{X_s}{((K_x(X_{bh}) + X_s)};$$

The parameter values are

$$K_{LA} = 4; K_{NH} = 1; K_{NO} = 0.5; K_{OA} = 0.4; K_{OH} = 0.2; \; \mathrm{K_S} = 20; K_X = 0.03; S_{ND,in} = 9; S_{NH,in} = 15;$$
$$S_{NO,in} = 1; S_{Omax} = 10; S_{s,in} = 200; X_{BA,in} = 0; X_{BH,in} = 0; \; \mathrm{S}_{o,in} = 2;$$
$$X_{ND,in} = 0; X_{p,in} = 0; X_{s,in} = 100; \; \mathrm{Y_A} = 0.24; Y_H = 0.67; ba = 0.05; bh = 0.22;$$
$$fp = 0.08; I_{xb} = 0.086; I_{xp} = 0.06; \; \mathrm{K}_a = 0.081; K_h = 3; \eta_g = 0.8; \eta_h = 0.4;$$

The variables $S_S, X_S, X_{BH}, X_{BA}, S_O, S_{NO}, S_{NH}, S_{ND}, X_{ND}$ represent the concentrations of readily biodegradable soluble substrate, slowly biodegradable particulate substrate, active heterotrophic particulate mass, active autotrophic particulate mass, soluble oxygen, soluble nitrate and nitrite nitrogen, soluble ammonium nitrogen, soluble biodegradable organic material, and particulate biodegradable organic nitrogen.

Bifurcation analysis

The MATLAB software MATCONT is used to perform the bifurcation calculations. Bifurcation analysis deals with multiple steady-states and limit cycles. Multiple steady states occur because of the existence of branch and limit points. Hopf bifurcation points cause limit cycles. A commonly used MATLAB program that locates limit points, branch points, and Hopf bifurcation points is MATCONT (Dhooge Govearts, and Kuznetsov, 2003[8]; Dhooge Govearts, Kuznetsov, Mestrom and Riet, 2004[9]). This program detects Limit pointsm(LP), branch points(BP), and Hopf bifurcation points(H) for an ODE system

$$\frac{dx}{dt} = f(x, \alpha) \qquad (2)$$

$x \in R^n$ Let the bifurcation parameter be α Since the gradient is orthogonal to the tangent vector,

The tangent plane at any point $w = [w_1, w_2, w_3, w_4, ...w_{n+1}]$ must satisfy

$$Aw = 0 \qquad (3)$$

Where A is

$$A = [\partial f / \partial x \quad | \partial f / \partial \alpha] \qquad (4)$$

where $\partial f / \partial x$ is the Jacobian matrix. For both limit and branch points, the matrix $[\partial f / \partial x]$ must be singular. The n+1 th component of the tangent vector w_{n+1} = 0 for a limit point (LP)and for a branch point (BP) the matrix $\begin{bmatrix} A \\ w^T \end{bmatrix}$ must be singular. At a Hopf bifurcation point,

$$\det(2 f_x(x, \alpha) @ I_n) = 0 \qquad (5)$$

@ indicates the bialternate product while I_n is the n-square identity matrix. Hopf bifurcations cause limit cycles and should be eliminated because limit cycles make optimization and control tasks very difficult. More details can be found in Kuznetsov (1998[10]; 2009[11]) and Govaerts [2000] [12]

Results and Discussion:

The bifurcation analysis on the ASM1 model revealed the existence of two branch points at

$(S_S, X_S, X_{BH}, X_{BA}, S_O, S_{NO}, S_{NH}, S_{ND}, X_{ND}, d)$ values of (200, 56.179, 0, 0,9.65, 1, 15, 9, 0, 0.179) and (200.000000 56.179, 0, 0,9.36, 1, 15, 9, 0,0.343). These branch points are indicated in Figure. 1.

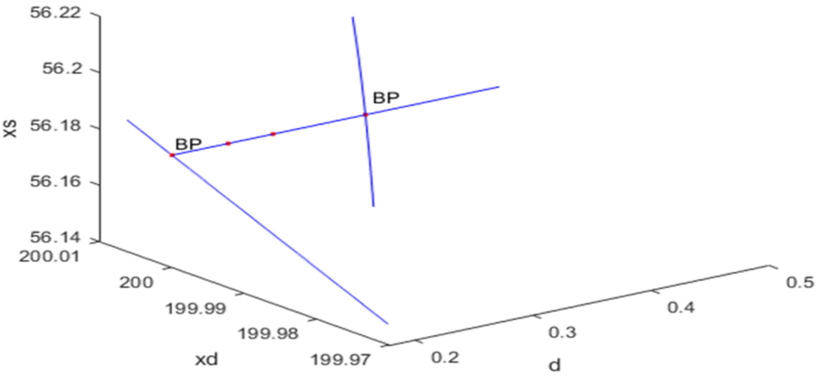

Figure 1: *Branch points for ASM1 model*

The presence of the branch points is beneficial because they allow the MNLMPC calculations to attain the Utopia solution for several objective functions.

Three MNLMPC calculations were performed. In the first case, the particulate variables (active heterotrophic particulate mass, active autotrophic particulate mass, and particulate biodegradable organic nitrogen) were minimized. In this case, $\sum_{t_i=0}^{t_i=t_f} X_{BH}(t_i), \sum_{t_i=0}^{t_i=t_f} X_{BA}(t_i), \sum_{t_i=0}^{t_i=t_f} X_{ND}(t_i)$ was minimized individually and each of them led to a value of 0 . The overall optimal control problem will involve the minimization of $(\sum_{t_i=0}^{t_i=t_f} X_{BH}(t_i))^2 + (\sum_{t_i=0}^{t_i=t_f} X_{BA}(t_i))^2 + (\sum_{t_i=0}^{t_i=t_f} X_{ND}(t_i))^2$ was minimized subject to the equations governing the model. This led to a value of zero (the Utopia solution.

The various concentration profiles for this MNLMPC calculation are shown in Figs. 2a-2d.

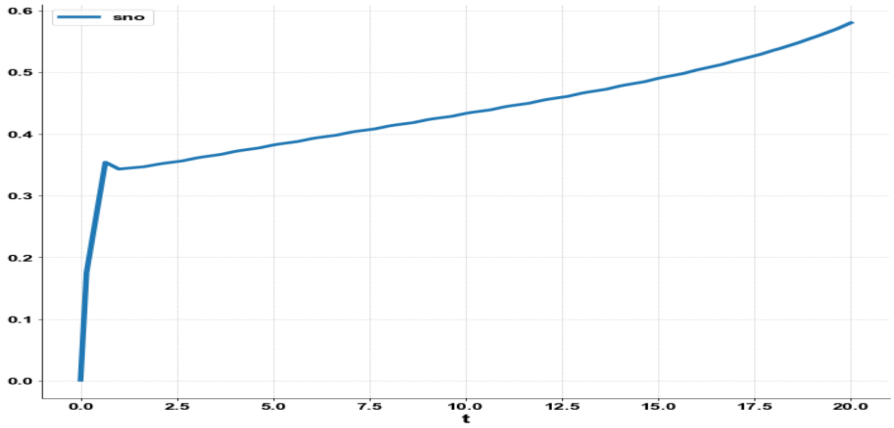

Figure 2a: *SNO profile MNLMPC particulate concentration minimization*

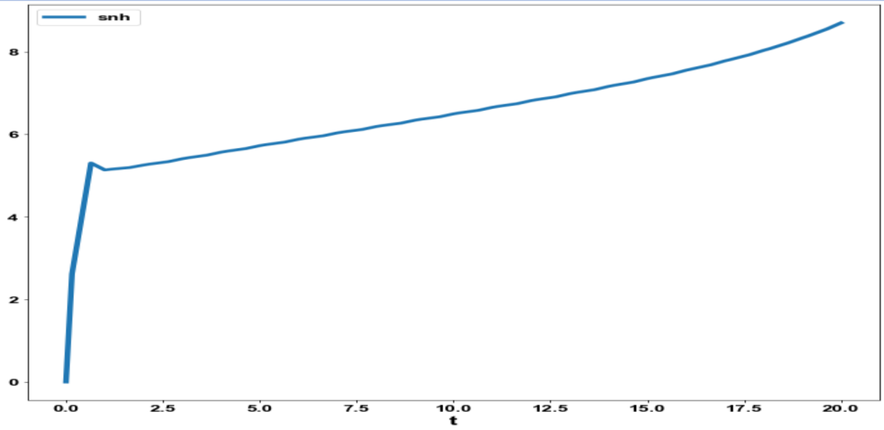

Figure 2b: *SNH profile MNLMPC particulate concentration minimization*

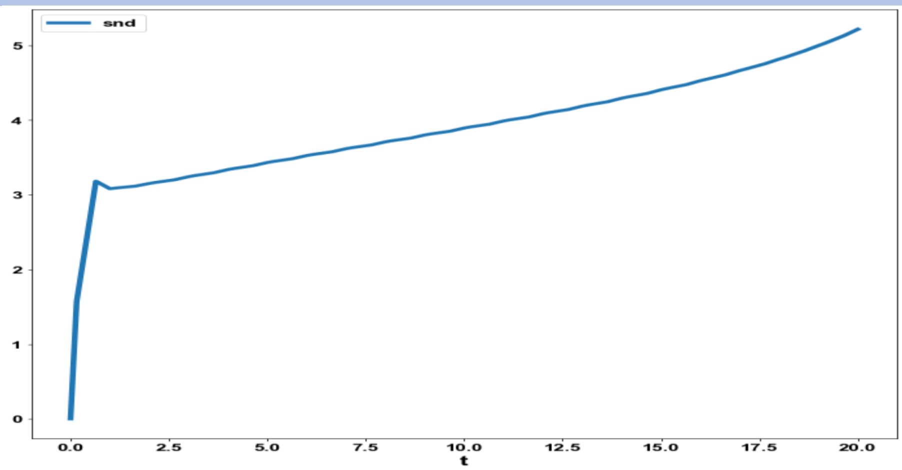

Figure 2c : *SNO profile MNLMPC particulate concentration minimization*

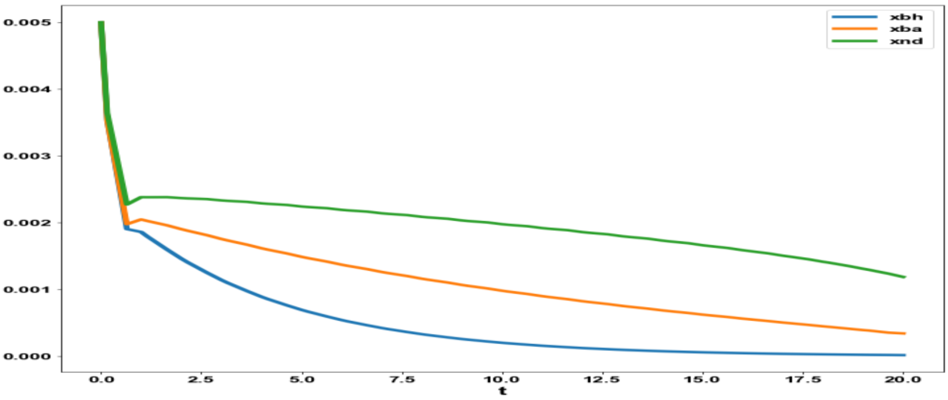

Figure 2d: *XBH, XBA, XND profile MNLMPC particulate concentration minimization*

The obtained control profile of s exhibited noise (Fig. 2e). This was remedied using the Savitzky-Golay Filter. The smoothed-out version of this profile is shown in Fig.2f.

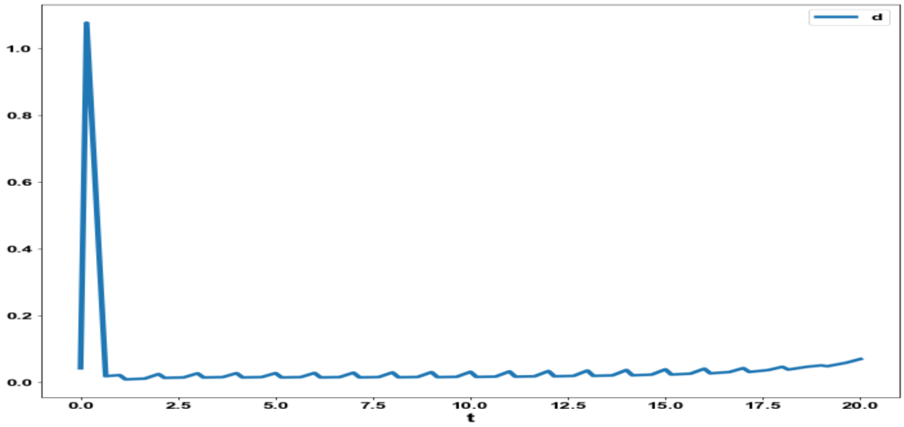

Figure 2e : *dilution rate MNLMPC particulate concentration minimization*

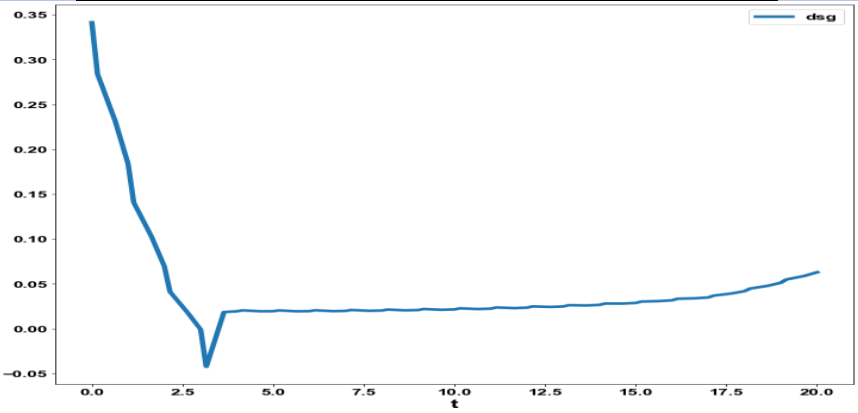

Figure 2f: *dilution rate (with Savitzky Golay filter) MNLMPC particulate concentration minimization*

In the second case, the variables representing the soluble materials(soluble nitrate and nitrite nitrogen, soluble ammonium nitrogen, and soluble biodegradable organic material) were minimized. In this case, $\sum_{t_{i=0}}^{t_i=t_f} S_{NO}(t_i), \sum_{t_{i=0}}^{t_i=t_f} S_{NH}(t_i), \sum_{t_{i=0}}^{t_i=t_f} S_{ND}(t_i)$ was minimized individually, leading to values of 0.4121, 4.722, and 0.019971. The overall optimal control problem will involve the minimization of $(\sum_{t_{i=0}}^{t_i=t_f} S_{NO}(t_i) - 0.4121)^2 + (\sum_{t_{i=0}}^{t_i=t_f} S_{NH}(t_i) - 4.722)^2 + (\sum_{t_{i=0}}^{t_i=t_f} S_{ND}(t_i) - 0.019971)^2$ was minimized subject to the equations governing the model. This led to a value of zero (the Utopia solution.

The various concentration profiles for this MNLMPC calculation are shown in Figs. 3a-3d.

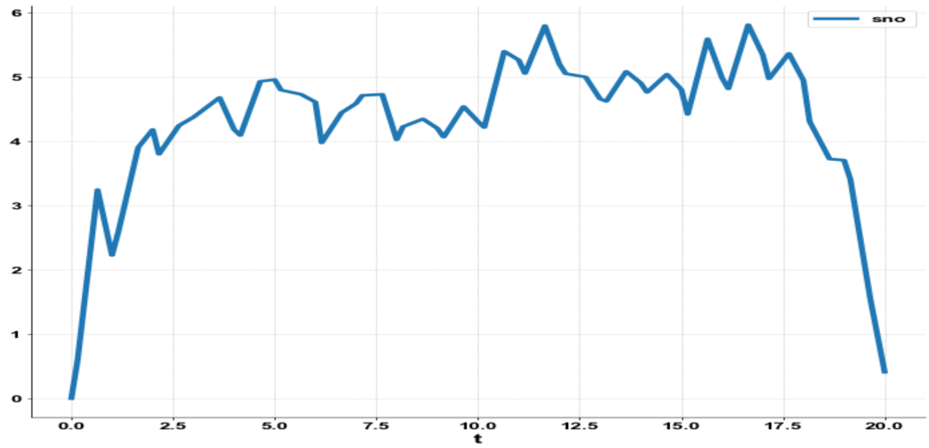

***Figure 3a** : SNO profile MNLMPC soluble material concentration minimization*

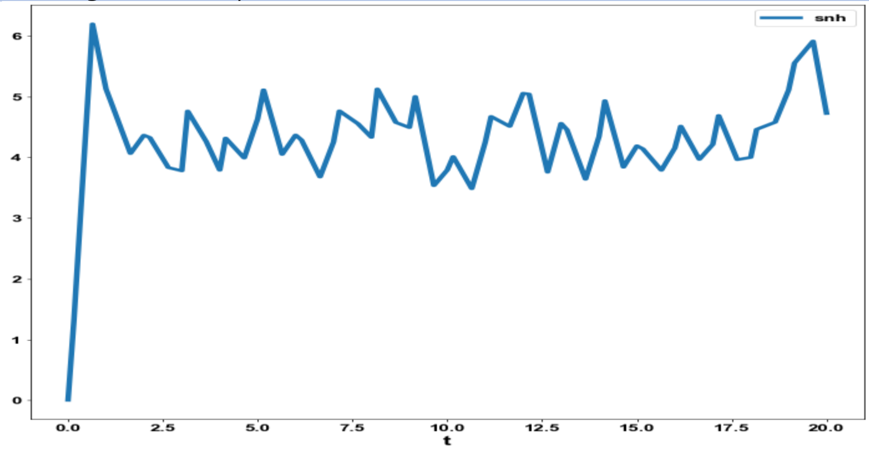

Figure 3b *: SNH profile MNLMPC soluble material concentration minimization*

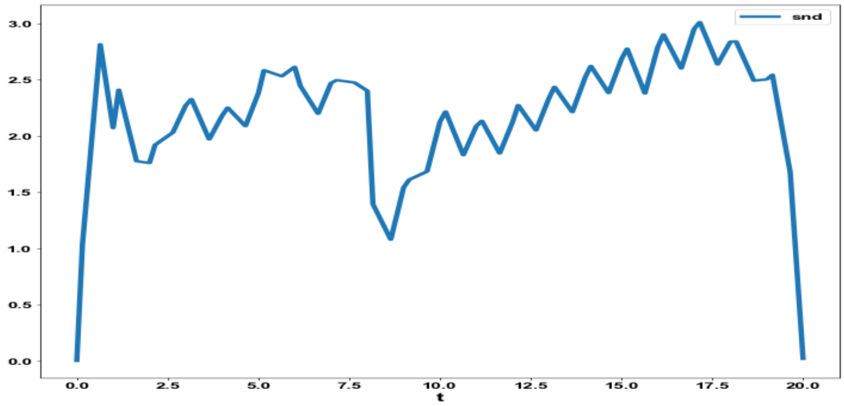

Figure. 3c: *SND profile MNLMPC soluble material concentration minimization*

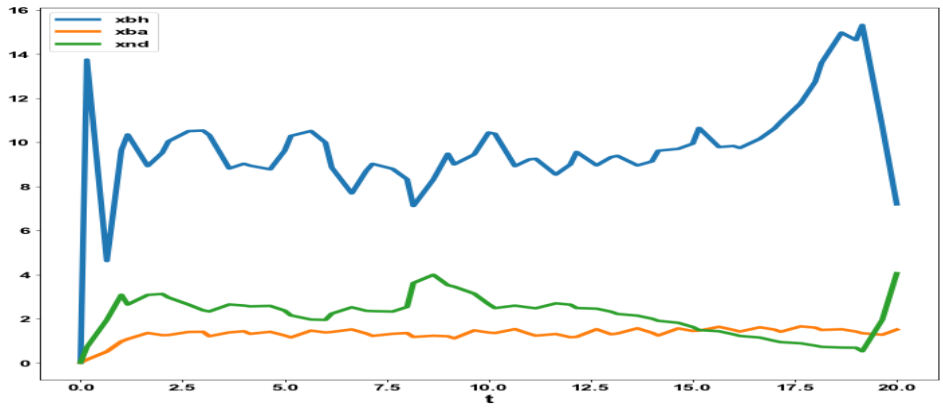

Figure 3d: XBH, XBA, XND profile MNLMPC soluble material concentration minimization

The obtained control profile of s exhibited noise (Fig. 3e). This was remedied using the Savitzky-Golay Filter. The smoothed-out version of this profile is shown in Fig.3f.

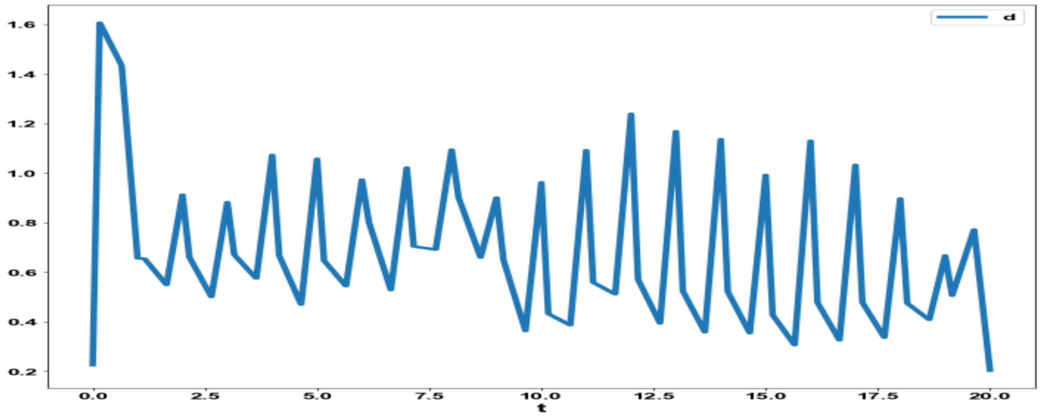

Figure. 3e: *dilution rate MNLMPC soluble material concentration minimization*

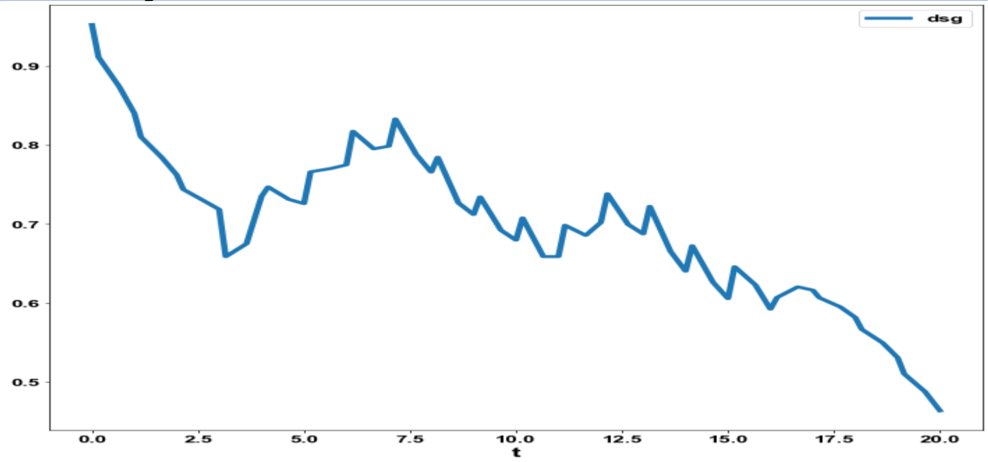

Figure. 3f : *dilution rate (with Savitzky Golay filter) MNLMPC soluble material concentration minimization*

In the third case, In the second case, the variables representing the soluble materials(soluble nitrate and nitrite nitrogen, soluble ammonium nitrogen, and soluble biodegradable organic material) and the particulate variables (active heterotrophic particulate mass, active autotrophic particulate mass, and particulate biodegradable organic nitrogen) were clubbed together as S_{total} and X_{total} . In this case, $\sum_{t_{i=0}}^{t_i=t_f} S_{total}(t_i), \sum_{t_{i=0}}^{t_i=t_f} X_{total}(t_i)$ was minimized individually, leading to values of 10.8079 and 0.01647 . The overall optimal control problem will involve the minimization of $(\sum_{t_{i=0}}^{t_i=t_f} S_{total}(t_i)-10.8079)^2+(\sum_{t_{i=0}}^{t_i=t_f} X_{total}(t_i)-0.01647)^2$ was minimized subject to the equations governing the model. This led to a value of zero (the Utopia solution). The various concentration profiles for this MNLMPC calculation are shown in Figs. 4a-4d. The obtained control profile of s exhibited noise (Fig. 4e). This was remedied using the Savitzky-Golay Filter. The smoothed-out version of this profile is shown in Fig.4f.

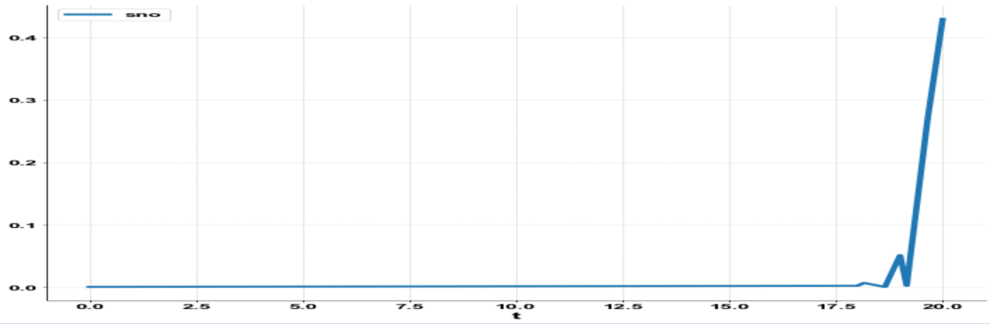

Figure. 4a: *SNO profile MNLMPC X and S concentration minimization*

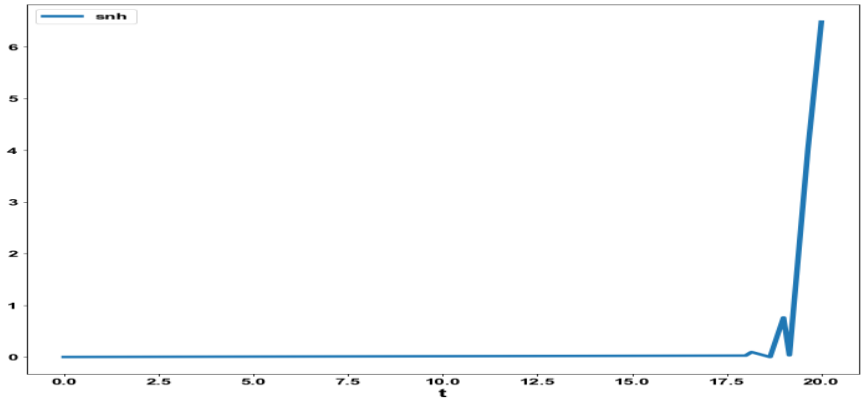

Figure 4b: *SNH profile MNLMPC X and S concentration minimization*

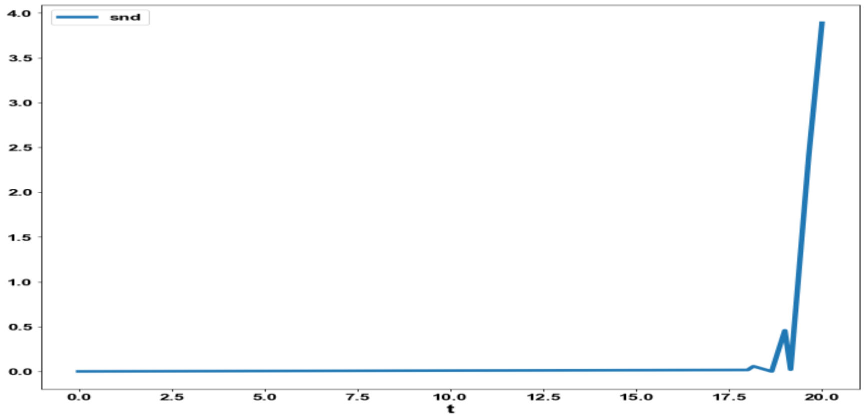

Figure 4c : *SND profile MNLMPC X and S concentration minimization*

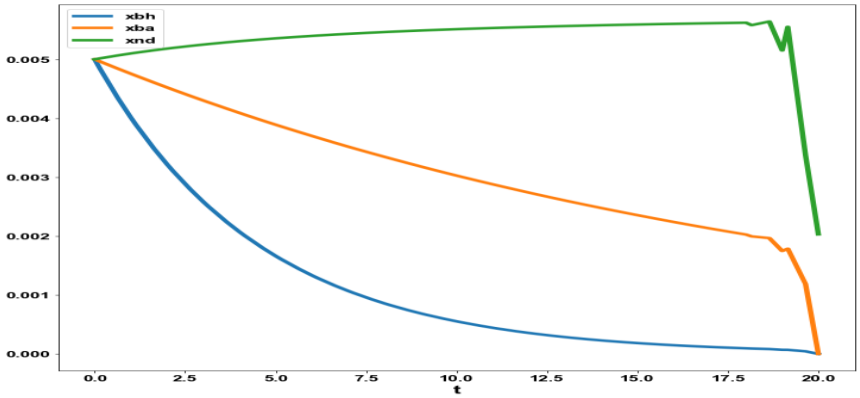

Figure 4d: *XBH, XBA, XND profile MNLMPC X and S concentration minimization*

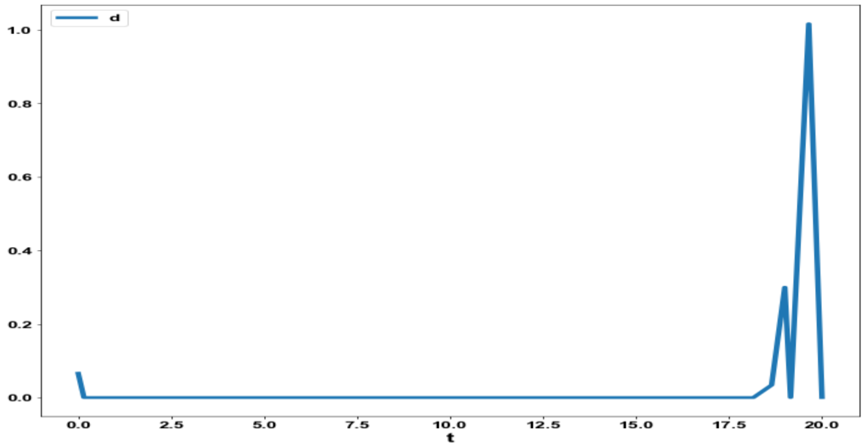

Figure. 4e: *dilution rate MNLMPC X and S concentration minimization*

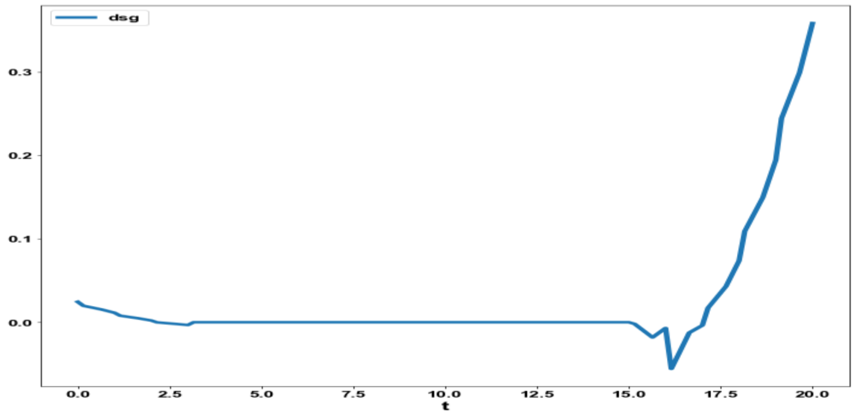

Figure 4f : *dilution rate (with Savitzky Golay filter) MNLMPC X and S concentration minimization*

In all the cases, the MNLMPC calculations converged to the Utopia solution, validating the analysis of Sridhar (2024), which showed that the presence of a limit or branch point enables the MNLMPC calculations to reach the best possible (Utopia) solution.

Conclusions:

Bifurcation analysis and Multiobjective nonlinear model predictive control calculations were performed on the activated sludge model (ASM1). The bifurcation analysis revealed the existence of branch points. The branch points (which produced multiple steady-state solutions originating from a singular point) are very beneficial as they caused the multiojective nonlinear model predictive calculations to converge to the Utopia point (the best possible solution) in both models. A combination of bifurcation analysis and multiobjective nonlinear model predictive control for the activated sludge model (ASM1) is the main contribution of this paper.

Data Availability Statement

All data used is presented in the paper

Conflict of interest:

The author, Dr. Lakshmi N Sridhar has no conflict of interest.

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